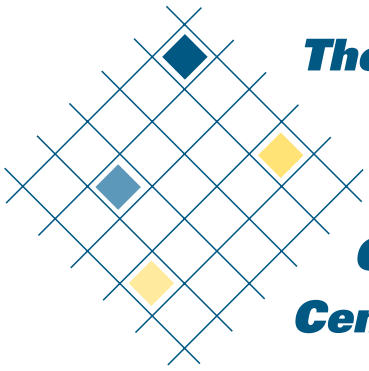

The Changing Mathematics Curriculum: *An Annotated Bibliography*

Third Edition
April 2005

The K–12 Mathematics Curriculum Center

The K–12 Mathematics Curriculum Center (K–12 MCC) supports school districts as they build effective mathematics education programs using curricula that align with the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Principles and Standards for School Mathematics* (2000).

The K–12 MCC offers a variety of products and services to assist mathematics teachers and administrators. Our seminars address selecting and implementing new curricula, designing professional development and support, aligning curriculum with assessment, and examining leadership in curricular change. Our other resources include:

- *Curriculum Summaries*
- *Perspectives on Curricular Change: Interviews with Teachers, Administrators, and Curriculum Developers* who have worked with 12 National Science Foundation (NSF)-funded *Standards*-based curricula
- *Choosing a Standards-Based Mathematics Curriculum*, a guide that suggests processes for selecting and implementing curricula
- a series of *Issues Papers* that explores contemporary mathematics education issues
- *Consisdering New Mathematics Curricula DVD*

Information and links to resources about *Standards*-based curricula and their implementation can be found at the K–12 Mathematics Curriculum Center website:

http://www.edc.org/mcc

Please contact the K–12 MCC for information on how we can either offer you direct support, point you to relevant resources and materials, or connect you with educators experienced in curriculum selection and implementation.

For more information about our materials and services please contact:

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About This Publication

This publication, an annotated bibliography of articles relevant to *Standards*-based mathematics curriculum reform, is intended as a resource for educators and communities considering the selection and implementation of a *Standards*-based mathematics curriculum. It also may assist individuals who are interested in learning about the student achievement, classroom practices, and implementation challenges associated with the use of *Standards*-based materials.

When gathering resources for this publication, the K–12 Mathematics Curriculum Center staff reviewed articles that either addressed important issues in mathematics curriculum change or shared experiences, views, and data relevant to the selection and use of 12 comprehensive mathematics programs based on the NCTM *Standards* and developed with support from the NSF. These programs are listed on the back cover of this publication.

This bibliography does not attempt to be comprehensive and certainly does not include every article written about mathematics curriculum reform. Rather, the goal was to select articles that would increase readers' understanding of *Standards*-based mathematics curricula and their use in classrooms.

Most of the articles are peer-reviewed and were either published in journals or collections, or presented as papers at conferences. They are organized into four categories: characteristics of *Standards*-based mathematics curricula, research regarding the impact of these programs on student achievement, professional development, and issues that arise during implementation.

Selections beginning with the word "Abstract" were written by the original authors or publishers. If an article was accessible online at the time of publication, its URL is listed. Some of these articles can be read or downloaded free of charge; others are available to subscribers and/or can be purchased for a nominal fee.

This publication is also available on our website at:

http://www.edc.org/mcc

This book was developed by the K–12 Mathematics Curriculum Center at Education Development Center, Inc. The work was supported by National Science Foundation Grant No. ESI-0137826. Opinions expressed are those of the authors and not necessarily those of the Foundation.

National Grade-level Implementation Centers

In addition to the K–12 Mathematics Curriculum Center, there are three grade-level specific implementation centers supporting these curricula.

Alternatives for Rebuilding Curricula (ARC)

Contact: Sheila Sconiers
COMAP, Inc.
Suite 210
57 Bedford Street
Lexington, MA 02420
Phone: (800) 772-6627, ext. 50
Fax: (781) 863-1202
E-mail: arccenter@mail.comap.com
Website: http://www.comap.com/elementary/projects/arc

The ARC Center is a collaboration between the Consortium for Mathematics and Its Applications (COMAP) and the three NSF-supported elementary mathematics curriculum projects, *Investigations in Number, Data, and Space* (TERC); *Math Trailblazers* (University of Illinois at Chicago); and *Everyday Mathematics* (University of Chicago). The Center promotes the wide-scale and effective implementation of reform elementary mathematics curricula.

Show-Me Center (National Center for *Standards*-based Middle Grades Mathematics Curricula)

Contact: Barbara Reys
University of Missouri
303 Townsend Hall
Columbia, MO 65211
Phone: (573) 884-2099
Fax: (573) 882-4481
E-mail: showmecenter@missouri.edu
Website: http://showmecenter.missouri.edu

The Show-Me Project is dedicated to providing information and assistance to schools considering and/or in the process of implementing *Standards*-based curriculum reform at the middle grade levels (grades 6–8). The Project is a partnership of curriculum developers and professional development staff at: the Show-Me Center at the University of Missouri–Columbia; the Math in Context Satellite Center at the University of Wisconsin; the Connected Mathematics Project Satellite Center at Michigan State University; the MathScape Satellite Center at Education Development Center; and the MATH Thematics Satellite Center at the University of Montana.

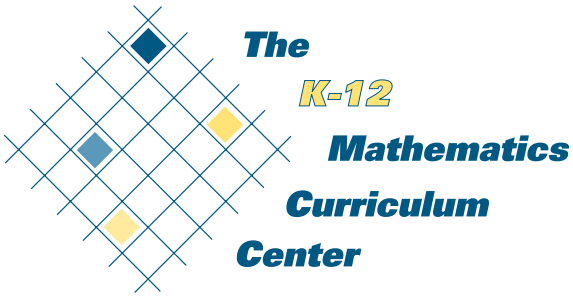
Project staff provide assistance to state and district personnel through teacher institutes, curriculum conferences, and teacher leadership conferences. The Project provides information, materials to assist district implementation, and an online curriculum showcase at its website. It also responds to requests for services and information needed by local district personnel.

COMPASS (Curricular Options in Mathematics Programs for All Secondary Students)

Contact: Eric Robinson
Ithaca College
306 Williams Hall
Ithaca, NY 14850
Phone: (800) 688-1829
(607) 274-1513
Fax: (607) 274-3054
E-mail: compass@ithaca.edu
Website: http://www.ithaca.edu/compass

The COMPASS project is a secondary school implementation project funded in part by the National Science Foundation. COMPASS consists of six sites: a satellite site for each of the five secondary-level curricula and a generic central site.

The primary function of the central site is to inform schools, districts, teachers, parents, administrators, state offices, and other groups about these innovative curricula and aid in the first general phase of selection and implementation. It also coordinates requests for additional information and assistance from the satellite sites. Each satellite provides additional information, implementation assistance, and professional development opportunities focused on its curricula to these same constituencies.



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Characteristics of Standards-Based Mathematics Curricula

Whether their concern is mathematics for kindergarteners (Isn't two plus two still four?), middle school students (Algebra—how has that changed?), or adolescents (What about calculus?), stakeholders—parents, teachers, administrators, school board members, and students—want to know what reform-oriented, Standards-*based curricula are all about. This section addresses stakeholder questions by providing articles and research related to the rigorous mathematics embedded in curricula that were developed under the National Science Foundation's (NSF) sponsorship and aligned with the* Principles and Standards *put forth by the National Council of Teachers of Mathematics (NCTM). Also included are articles about researched-based methods of teaching and learning mathematics, as well as information to assist stakeholders when investigating, selecting, and adopting* Standards-*based curricula.*

Curricular Implications of the NCTM *Standards*

Alper, L., Fendel, D., Fraser, S., & Resek, D. (1997). Designing a high school curriculum for all students. *The American Journal of Education, 106*(1), 148–178.

Four developers of the *Interactive Mathematics Program* (IMP) discuss the need for mathematics education to reach all students through a shift "from the learning of procedures to the solving of complex problems," so that today's students will be prepared for tomorrow's workforce. The principles of curriculum development that guide the IMP high school curriculum are discussed: 1) Students must feel at home in the curriculum (accomplished through concrete introduction of ideas, minority representation in the curriculum, the social environment of the classroom, and involving the community); 2) Students must feel personally validated as they learn (by creating an atmosphere of respect and validation); 3) Students must be actively involved in their learning (by adapting the curriculum for active involvement, creating individualized instruction, and providing for individual differences); and 4) Students need a reason for doing problems (accomplished by providing real-life contexts, capturing students' imagination, and tapping their intellectual curiosity). Examples of exercises from the IMP curriculum illustrate each of these principles. The article then turns to evaluate this approach by looking at the achievement of IMP students, including their performance on standardized tests and beyond. In conclusion, the article addresses political concerns that oppose *Standards*-based curricula, and presents areas for future research.

Burrill, G. (1998). Changes in your classroom: From the past to the present to the future. *Mathematics Teaching in the Middle School, 4*(3), 184–190. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME1998-11-583a&from=B>.

Gail Burrill explores implications of the NCTM *Standards* on mathematics curriculum reform by taking a look at where we have come from (in terms of student mathematical performance and understanding), where we are today (in terms of statistics, referencing the Third International Mathematics and Science Study (TIMSS); myths about mathematics; the challenge of tracking; and the mathematics that children like to do) and where we are headed (with technology creating jobs that require greater and different mathematical understanding). Burrill sees the need to "build a curriculum that flows throughout the grades into one coherent whole, in which students are expected to have a common knowledge base by a given grade level, and in which teachers act on this expectation." A curriculum designed in this vein, Burrill argues, will 1) cut down on the "repeat and remediation" cycle in which today's mathematics education is often embedded, 2) allow for a broader and more useful base of mathematics to be explored in the classroom, and 3) make mathematics consistent across grade levels nationally. The article cites examples of problems that both develop mathematical concepts and allow them to be practiced and applied. Burrill concludes by addressing challenges that face teachers, teacher educators, supervisors, mathematics education researchers, and mathematicians.

Curcio, F. (1999). Dispelling myths about reform in school mathematics. *Mathematics Teaching in the Middle School, 4*(5), 282–284. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-02-282a&from=B>.

This article aims to dispel five myths that surround mathematics reform efforts and *Standards*-based curricula: 1) Basic computation is ignored; 2) Answers that are close to correct are good enough; 3) Only one right way exists to teach mathematics; 4) Textbooks identified as "standards-based" support reform efforts; 5) No research is available to support reform efforts. Curcio provides arguments and information to refute all five misunderstandings. The author cites the rationale behind the NCTM *Standards* and explains the roles of new pedagogy, new coverage of mathematics topics, and the role of technology in mathematics instruction. The article advocates against allowing myths such as these to interfere with discussions about improving mathematics education.

Fennell, F. & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7*(5), 288. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=TCM2001-01-288a&from=B>.

The article opens with a vignette of four third-graders working on a story problem. Each child uses a different computation strategy and different tools to represent his or her thinking about the problem. The authors build the article from this, explaining that representation is the way a student shows the steps he or she takes to solve a problem. Different children may choose different strategies for representation: drawing pictures, creating mental images, using manipulatives or computer models, or writing equations. The authors stress that good representations should accurately reflect a student's thinking about and understanding of the problem. In the article, the authors explore the use of representations in the classroom, highlighting the importance of modeling and of exposing students to a variety of representations. They describe students' reactions to representations of various mathematical concepts, including measurement, area and perimeter, and then identify what teachers can learn from each example.

Goldsmith, L. & Mark, J. (1999). What is a *Standards*-based mathematics curriculum? *Educational Leadership, 57*(3), 40–45. URL: <http://pdonline.ascd.org/pd_online/childmath/el199911_goldsmith.html>.

Goldsmith and Mark describe the many features that distinguish *Standards*-based mathematics curricula from traditional programs, focusing on new views of mathematics learning. Evaluating and choosing curriculum, the notion of "mathematics for all," and the constructivist nature of *Standards*-based mathematics programs are also addressed. This article is based on and adapted from ideas presented in *Choosing a Standards-Based Mathematics Curriculum*, by Lynn Goldsmith, June Mark, and Ilene Kantrov (Heinemann, 2000). This particular chapter is also available at http://www2.edc.org/mcc/mguide.asp.

Hart, E. & Stewart, J. (1998). Reflections on high school reform and implications for middle school. In L. Leutzinger (Ed.), *Mathematics in the Middle* (pp. 65–73). Reston, VA: National Council of Teachers of Mathematics.

This article describes the current reform in high school mathematics and discusses some of the implications for middle school reform. Common themes of high school reform include: more integration among mathematical topics, building connections to the real world, emphasizing active learning, the importance of assessment as a component of reform, and making mathematics accessible to all students. As described in this article, reform means a dramatic change in content, pedagogy, and assessment. These changes have implications for what mathematics is taught, how it is taught, and how it is assessed in middle school. The article reviews changes in four high school content strands (algebra, geometry and trigonometry, statistics and probability, and discrete mathematics) and discusses ways in which these strands can be reformed in middle school classrooms. Connections between teaching and assessment in high school and middle school mathematics curricula are also addressed.

Hirsch, C. & Coxford, A. (1997). Mathematics for all: Perspectives and promising practices. *School Science and Mathematics, 97*(5), 232–241.

The *Core-Plus Mathematics Project* (published as *Contemporary Mathematics in Context*) is one of a set of comprehensive curriculum development projects that, in 1992, were awarded five-year grants by the National Science Foundation to design, evaluate, and disseminate innovative high school curricula that interpret and implement the recommendations of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* and the *Professional Standards for Teaching Mathematics*. This article describes Core-Plus perspectives on a new curriculum organization for high school mathematics, identifies implications of these perspectives for promoting access and equity for all students, and reports some of the supporting oral data from an ongoing formative evaluation of the curriculum.

Reys, B. & Bay-Williams, J. (2003). The role of textbooks in implementing the curriculum principle and the learning principle. *Mathematics Teaching in the Middle School, 9*(2), 120–125. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2003-10-120a&from=B>.

This article serves as a summary of the curriculum and learning principles from NCTM's *Principles and Standards for School Mathematics*. The authors provide an overview of both principles, explain what each advocates, and suggest ways to apply the principles to practice in schools. The article also compares a sample exercise from a traditional textbook with one from a textbook developed to comply with the NCTM *Standards*. In closing, the authors make recommendations for effective ways of reviewing textbooks so as to judge their alignment with the *Principles and Standards*.

Reys, B., Robinson, E., Sconiers, S., & Mark, J. (1999). Mathematics curricula based on rigorous national standards: What, why and how? *Phi Delta Kappan, 80*(6), 454–456. URL: <http://www.ithaca.edu/compass/whatwhy.htm>.

The authors state that teachers, administrators, and parents need to become informed about the unique characteristics of the mathematics curricula based on the NCTM *Standards* and about the support structures that are being established to make it easier for schools to adopt them. This information is vital so that each district can make an informed decision about the mathematics curricula that best suit its needs.

Reys, R. (2001). Curricular controversy in the math wars: A battle without winners. *Phi Delta Kappan, 83*(3), 255–258. URL: <http://www.pdkintl.org/kappan/k0111rey.htm>.

In this short article, Reys outlines hurdles to innovation in mathematics instructional materials. In a profit-driven textbook market, with politically-driven textbook adoption practices that vary from state to state, publishers try to make their products marketable to the broadest possible audience. This type of climate has been a difficult one for*Standards*-based curriculum materials that were developed to improve U.S. students' mathematics learning and understanding. Reys suggests, however, that the dialogue created by these new materials has the potential to be productive and constructive, rather than inflammatory.

Romberg, T. (1997). The influence of programs from other countries on the school mathematics reform curricula in the United States. *American Journal of Education, 106*(1), 127–147.

Abstract: For too long, most Americans have been provincial in their thinking about schooling in other countries. As contact with educators from other nations has increased, we have become aware of differences in the mathematics included in the school curriculum, in the methods used in teaching mathematics, and in the way student performance is assessed in other countries. Romberg claims that at a general level this growing awareness has influenced the policies underlying the current curriculum reform movement in the United States. This article looks at the influence of developments in other countries on those reform efforts and materials and, specifically, at one example of curriculum materials now being used in American schools, which was guided by research and reform in the Netherlands.

Schoen, H., Fey, J., Hirsch, C., & Coxford, A. (1999). Issues and options in the math wars. *Phi Delta Kappan, 80*(6), 444–453. URL: <http://www.ithaca.edu/compass/issues.htm>.

The authors of this article reassess the case for change in mathematics education and examine the objections of critics in light of recent research and evaluation evidence. The article discusses the "reform consensus" for K–12 mathematics, which was shaped by international study results, mathematics professionals, teaching and learning research, prospects of technology in the classroom, and teacher experience. Issues within this consensus are discussed: the content and process goals of an implied "national curriculum," approaches to teaching and learning, assessment of student learning, and prospects for reform. The *Contemporary Mathematics in Context* (Core-Plus) high school mathematics curriculum is used as an example to illustrate a realization of the *Standards*' agenda. The authors discuss the structure and underlying principles of the curriculum, citing examples of mathematics problems from the program. Criticism of mathematics education reform is discussed, followed by an overview of positive evaluation evidence from the national Core-Plus field test (36 schools in 12 states). To conclude, the authors address the question "to change or not to change?", encouraging the use of programs like Core-Plus and noting that more research evidence is needed to further reform efforts.

Schoenfeld, A. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25. URL: <http://www.aera.net/uploadedFiles/Journals_and_Publications/Journals/Educational_Researcher/3101/3101_Schoenfeld.pdf>.

Citing Robert Moses' statement that "mathematics education is a civil rights issue," this article discusses the need for mathematical literacy among all students for civic, social, and economic reasons. The author summarizes national mathematics education trends, as well as recent efforts in curriculum design to align instructional materials with NCTM's *Curriculum and Evaluation Standards* (1989) and *Principles and Standards* (2000). The author finds convergence in data from research about the impact of these new, innovative curricula, leading to several generalizations: 1) On tests of basic skills, no significant difference in performance is evident between students who learn from traditional or reform materials; 2) Students who learn from reform curricula consistently outperform students using traditional curricula on tests of conceptual understanding and problem-solving; 3) Some evidence shows that use of reform programs may narrow the performance gap between white students and under-represented minorities. The article then shifts to examine the central issues in making *Standards*-based mathematics instruction effective. Important factors include 1) high-quality curricula that are continually improved based on feedback and have a better-articulated vision for mathematics learning, 2) consideration of mathematics teaching as a profession requiring ongoing training and support, 3) using assessment as a method of informing instruction and fostering students' learning, and 4) maintaining stability in the development of mathematics education and creating mechanisms to support its evolution.

Silver, E. (2000). Improving mathematics teaching and learning: How can *Principles and Standards* help? *Mathematics Teaching in the Middle School, 6*(1), 20–23. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-09-20a&from=B>.

This short article highlights some of the goals for middle-grades students' mathematics learning that are highlighted in NCTM's *Principles and Standards for School Mathematics* (PSSM). The author presents a particular problem, the Circles-in-the-Square problem, as an example of how ideas that are mathematically important in the middle grades (proportionality, algebra, and geometry) are explored and connected within a single problem. He then delineates the goals of PSSM for student learning in

each of these topics at the middle grades, suggesting that these goals are a good basis for curriculum-focused conversations.

Trafton, P., Reys, B., & Wasman, D. (2001). *Standards*-based mathematics curriculum materials: A phrase in search of a definition. *Phi Delta Kappan, 83*(3), 259–264. URL: <http://www.pdkintl.org/kappan/k0111tra.htm>.

The authors first lay out a brief history of the NCTM *Standards* and the resulting term "*Standards*-based" and then highlight the characteristics of *Standards*-based mathematics instructional materials. The article explains that *Standards*-based materials differ from past mathematics curricula in that they are comprehensive and coherent; they develop ideas in depth; they promote sense-making; and they engage students and motivate their learning. Because of these differences, the use of *Standards*-based materials has many implications for change in teaching practice.

Pedagogy

Alper, L., Fendel, D., Fraser, S., & Resek, D. (1996). Problem-based mathematics—not just for the college bound. *Educational Leadership, 53*(8), 18–21.

The authors explore differences in the approach and structure of the *Interactive Mathematics Program* (IMP). The article begins with an example (a unit called "Meadows or Malls?") from IMP's problem-based mathematics curriculum, where students work as if they were city planners deciding how best to use public land. Problems such as these begin units and are often too difficult for students to solve right away; this opens students' thinking so that they pose questions, look for patterns, and make connections to mathematics they already know. The article also addresses parent concerns that *Standards*-based curricula do not contain enough repetition for students to master skills; it explains that problems encountered in IMP encourage students to discover and construct ideas, rather than merely memorize definitions, for more meaningful learning. To conclude, the authors address the "results" that can be seen from curricula like IMP by looking at studies that follow students' progress and learning in school and after graduation.

Alper, L., Fendel, D., Fraser, S., & Resek, D. (1995). What is it worth? *The Mathematics Teacher, 88*(7), 598–602.

This article discusses the mathematics teacher's changing role in choosing curriculum tasks that are aligned with the NCTM *Standards*, and how curriculum programs like the *Interactive Mathematics Program* (IMP) work to develop quality mathematical activities. The article reviews characteristics of meaningful tasks: tasks must be mathematically worthwhile, students' minds must be involved, and students must believe in the worth of the task (which involves real-life context, catching students' imagination, and tapping into their intellectual curiosity). The article cites some classroom examples from the IMP curriculum, and shows ways in which these activities can help teachers lead challenging *Standards*-based mathematics classes. Structuring the difficulty of tasks for different student populations, and successful classroom approaches are discussed.

Battista, M. (1999). The mathematical miseducation of America's youth: Ignoring research and scientific study in education. *Phi Delta Kappan, 80*(6), 424–433. URL: <http://www.pdkintl.org/kappan/kbat9902.htm>.

Battista responds to popular criticisms of the current mathematics education reform movement by claiming that critics "lack understanding both of the essence of mathematics, and of scientific research on how students learn mathematics." Battista reviews the current state of traditional mathematics education and the reasoning behind reform. Taking a look at how children construct mathematical knowledge, Battista sees a misalignment between how mathematics has traditionally been taught and how mathematics can be learned effectively. The "genuine issues in improving mathematics learning" addressed in this article are 1) the lack of adult mathematical knowledge, 2) the disregard of scientific practice within the field of education, 3) the fact that there are researchers who are non-specialists within the field of education, 4) the myth that "if mathematics is 'covered,' students will learn it," 5) the understanding of the process of testing, and 6) the dilutions and distortions of ideas underlying mathematics education reform. To conclude, Battista asserts that without extensive knowledge about the essence of mathematics and without understanding current research on how students learn mathematics, judgments made about how and what to teach are naive—and mostly wrong. He urges the educational community to act more responsibly with respect to mathematics curriculum decisions, demanding that decisions be consistent with relevant scientific findings by mathematics education specialists.

Carroll, W. & Porter, D. (1997). Invented strategies can develop meaningful mathematical procedures. *Teaching Children Mathematics, 3*(7), 370–374.

This article is based on interviews and observations that reveal insights into students' mathematical learning and understanding through their use of invented algorithms: "their own accurate solution procedures for multi-digit addition and subtraction, as well as for simple multiplication and division." The authors explore how invented procedures promote understanding through promotion of mathematics as a meaningful activity; the notion that different problems are best solved by different methods; and the observation that the natural tendencies of students often do not match standard algorithms. Ways described to encourage the invention of such

procedures are to 1) allow time for students to explore their own methods, 2) supply manipulatives that support students' thinking, 3) have children work to build strategies as well as knowledge, 4) provide meaningful contexts to inspire creative thinking, and 5) encourage children to work together and share strategies. The article presents examples of student strategies through descriptions and illustrations. To conclude, the authors remind readers that while invented algorithms are not always correct, it is important to build on children's thinking and allow them time to make sense of the mathematics.

Keiser, J. (2000). The role of definition. *Mathematics Teaching in the Middle School, 5*(8), 506–511. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-04-506a&from=B>.

This article explores the role of mathematical definitions in the middle grades. The author observed an introduction of the concept of angle with students using *Connected Mathematics* (CMP). A traditional approach to teaching the concept of angle and defining angles is contrasted to the approach used in CMP. The article concludes by reinforcing the idea that deeper conceptual understanding happens when students construct and discuss their own definitions and apply them to real contexts.

Kent, L. B. (2000). Connecting integers to meaningful contexts. *Mathematics Teaching in the Middle School, 6*(1), 62–66. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-09-62a&from=B>.

This article explores a variety of contexts that helped culturally-diverse fifth-graders learn concepts about and operations with integers. The author explains how the "Dry and Wet Numbers" unit in the *Mathematics in Context* curriculum relates "opposite numbers" (positive and negative integers) with specific contexts so that students build the understanding needed to work with positive and negative numbers. The author assesses the effectiveness of this sequence of integer lessons by noting students' progression from simply counting with integers to addition and subtraction with them. The article concludes by briefly describing other strategies used by *Mathematics in Context* to engage students' interest.

Kleiman, G., Tobin, D., & Isaacson, S. (1998). What should a middle school mathematics classroom look like? Watching the *Seeing and Thinking Mathematically* curriculum in action. In L. Leutzinger (Ed.), *Mathematics in the Middle* (pp. 120–128). Reston, VA: National Council of Teachers of Mathematics.

This article, written by the developers of the middle grades curriculum, *Mathscape: Seeing and Thinking Mathematically*, discusses the collaborative approach used to develop the curriculum by describing the four-year process of research, writing, and field-testing with students in classrooms. Ten "benchmarks of student success" in mathematics are explored: 1) Students demonstrate meaningful understanding of mathematical concepts; 2) Students' work and class discussions involve three components of mathematical discourse; 3) As they progress through the curriculum, students move from using their own approaches to more precise and powerful mathematical language and techniques; 4) Students share discoveries, discuss and write about mathematics, and engage in collaborative projects; 5) Students engage in creative mathematical work; 6) Students use computers to enhance their learning; 7) Students connect mathematics to other subjects and to their lives outside the classroom; 8) Teachers learn as well as teach; 9) Teachers adapt the curriculum to best meet the needs of their students; 10) All students are successful mathematics learners. Examples of problems from the curriculum are included.

Lappan, G. & Phillips, E. (1998). Teaching and learning in the *Connected Mathematics* program. In L. Leutzinger (Ed.), *Mathematics in the Middle* (pp. 83–92). Reston, VA: National Council of Teachers of Mathematics.

This article stems from work within the project that developed *Connected Mathematics* (CMP), a problem-centered, middle grades curriculum. The authors review the mathematics content strands discussed in the NCTM *Standards* and explain how CMP's goals for students and teachers are aligned with them. Instructional themes highlighted are 1) teaching for understanding, 2) making connections among mathematics topics and with other subjects, 3) inquiry and discovery through mathematical investigations, 4) use of representations, and 5) use of technology. The authors also discuss changes involved in developing a "classroom climate," including issues of assessment and a reform-based model of teaching that includes three phases: Launch, Explore, and Summarize. Issues that the developers faced when creating the curriculum are listed along with criteria they used for creating worthwhile mathematical tasks. (Several examples of tasks with student responses are included.) Finally, the article examines impact data and shares graphed results of CMP student achievement that reflect score improvement on ITBS tests and balanced assessments.

Meyer, M. (1997). *Mathematics in Context*: Opening the gates to mathematics for all at the middle level. *National Association of Secondary School Principals (NASSP) Bulletin, 81*, 53–59.

This article summarizes the middle grades mathematics curriculum *Mathematics in Context*, highlighting aspects of the program that differ from traditional programs. Such aspects include: an experimental instructional sequence so that students immediately interact with mathematics in a meaningful way; the use of informal strategies so that students can abstract and construct mathematical concepts for deeper understanding of them; the use of models and representations for lower levels of

abstraction; opportunities for students to interact and share strategies in order to understand and reflect upon diverse approaches to mathematics; and the intertwining of multiple learning strands so that students understand mathematics as a continuum of ideas and not as isolated algorithms.

Meyer, M., Decker, T., & Querelle, N. (2001). Context in mathematics curricula. *Mathematics Teaching in the Middle School*, *6*(9), 522–527. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2001-05-522a&from=B>.

This article explains the increased use of context in *Standards*-based mathematics curricula and then focuses on the role that context plays in teaching and learning mathematics. Using examples from five *Standards*-based curricula, the authors describe five different roles for context: motivating students to explore new mathematics, offering students opportunities to apply mathematics, serving as a source of new mathematics, suggesting a source for problem-solving strategies, and providing students with models to increase their understanding. In addition, the authors identify characteristics of contexts that support student understanding and positively impact learning. Through the exploration of the roles context plays in learning as well as the characteristics that distinguish high-quality contexts, the authors develop criteria to assess uses of context in mathematics education.

Noble, T., Nemirovsky, R., Tierney, C.,& Wright, T. (1999). The way things change. *Hands On!*, *22*(1), 14–17. URL: <http://www.terc.edu/handsonIssues/s99/s99.pdf>.

This article describes an exercise from a fifth-grade unit on change in the *Investigations in Number, Data, and Space* curriculum. In this exercise, students are asked to represent a "travel" story using three different tools: Cuisenaire rods and meter sticks; a numeric table; and a computer program. The article illustrates how multiple explorations of a problem can lead to increased conceptual development and how different approaches can contribute to students' overall understanding.

Robinson, E., Robinson, M., & Maceli, J. (2000). The impact of *Standards*-based instructional materials in the classroom. In M. Burke & F. Curcio (Eds.), *Learning Mathematics for a New Century: 2000 Yearbook* (pp. 112–126). Reston, VA: National Council of Teachers of Mathematics.

The article describes features of curriculum programs developed in response to NCTM's *Curriculum and Evaluation Standards*, and the impact these features can have on students and teachers in the classroom. With examples from several comprehensive secondary mathematics curricula, the article examines the philosophical focus of these programs, as well as instructional strategies fostered in their use. For instance, in these programs, algorithms are considered tools that result from a thought process or points from which further mathematical thinking can proceed. Contexts are used to set mathematics in real-world situations and develop mathematical understanding, and mathematical topics are integrated within problems and units. In addition, the article discusses differences in the content of these materials as compared with their more traditional counterparts, as well as the implications for teachers to understand concepts of statistics and probability, geometry, calculus, and algebra and functions at all grade levels, as well as some discrete mathematics at the middle and high school levels. Finally, it explains the use of technology within these curricula as a tool for learning and seeing mathematics concepts. The article closes by pointing out that within these curricula that there are many different ways to construct effective mathematics learning across topics.

Russell, S. J. (1996). Changing the elementary mathematics curriculum: Obstacles and challenges. In D. Zhang, T. Sawada, & J. P. Becker (Eds.), *Proceedings of the China-Japan-U.S. Seminar on Mathematics Education* (pp. 174–189). Carbondale, IL: Board of Trustees of Southern Illinois University. URL: <http://investigations.terc.edu/relevant/ChangingElemMath.html>.

Russell describes and interprets scenarios that illustrate a shift from "disembodied numbers and operations" to "more realistic" problems in elementary school mathematics. Russell compares the shift in the ways educators approach how children learn best to the "old style" of elementary mathematics. Rather than just teaching skills, new methods advocate a focus on mathematical thinking as students work together, consider their own reasoning and the reasoning of other students, communicate about mathematics (orally, in writing, through pictures, etc.), carry out one or two problems thoughtfully during class, and use multiple strategies to check their work. New curricula focus on developing fluency in approaching, evaluating, and communicating mathematical ideas, as well as enjoying and appreciating them. Russell explores uses of new curricula as effective models for teacher development, and as powerful pedagogical shifts that emphasize developing a student's mathematical frame of mind.

Content

Billstein, R. (2004). You are cleared to land. *Mathematics Teaching in the Middle School, 10*(1), 18–22. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2004-08-18a&from=B>.

How are the problems found in *Standards*-based curricula created, developed, researched, and refined? Billstein, an author of *MATH Thematics*, one of the five middle-grades curricula funded for development by the National Science Foundation, presents a problem typical to *MATH Thematics* and tracks its journey en route to being used in the classroom. Beginning as an interesting question posed by the author and a colleague as they waited for a delayed airplane departure (Is there a pattern for numbering airport runways?), the problem's mathematical content is extracted, appropriate measurement devices are selected, diagrams are drawn, conjectures are made, and open-ended questions are devised to elicit students' mathematical thinking. Billstein found that by actively using the content, strategies, and tools of *Standards*-based mathematics his students became more mathematically aware thinkers. The article, which includes activities, extensions, and selected answers, gives a good description of what problem-based learning entails, and by comparison, how it differs from the problem-solving activities often amended to traditional texts.

Burrill, G. (1996). Data analysis in the United States: Curriculum projects. In B. Phillips (Ed.), *Papers on Statistical Education* (pp.15–26). Hawthorn, Australia: Swinburne.

Burrill reviews the history of data analysis and statistics content in the American K–12 mathematics curriculum, citing *Everyday Mathematics; Investigations in Number, Data, and Space; Mathematics in Context;* and *Contemporary Mathematics in Context* as examples of *Standards*-based curricula that feature data analysis as a content strand. Trends in the use of data analysis in these programs include: the integration of data analysis techniques to provide different approaches to standard concepts; the development of number sense through working with real data, interpretation of numbers, and numerical summaries; and data-collecting situations that provide practice for computation and manipulation skills. Burrill also reviews statistics content (graphing, plotting, and the critical thinking that surrounds these tasks), ideas of sampling and representativeness of data, the articulation of these concepts across grade levels, and numerical and statistical literacy.

Cuoco, A., Goldenberg, E. P., & Mark, J. (1997). Habits of mind: An organizing principle for mathematics curriculum. *Journal of Mathematical Behavior, 15*(4), 375–402. URL: <http://www.edc.org/MLT/ConnGeo/HOM.html>.

Abstract: By emphasizing the ways of thinking that are essential in mathematics, one can design mathematics courses that simultaneously serve the needs of students who will go on to advanced mathematical study and students who will not. The authors address a series of mathematical "habits of mind," arguing that students should be pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, and guessers. Using mathematical examples, the authors discuss mathematical approaches to things, and how geometers and algebraists approach their world. Materials for teaching and learning provide students with problems and activities to develop these habits of mind and put them into practice.

Harris, K., Marcus, R., McLaren, K., & Fey, J. (2001). Curriculum materials supporting problem-based teaching. *School Science and Mathematics*, 101(6), 310–318. URL: <http://static.highbeam.com/s/schoolscienceandmathematics/october012001/curriculummaterialssupportingproblembasedteaching/>.

How does the problem-based *learning* of reform curricula differ from the problem-solving *activities* that have been added to traditional texts? The authors begin by describing the nature of tasks that have a high potential for student learning of challenging mathematics, and then take the reader through three problem-based learning lessons from middle and high school reform curricula that address the circumference and area of circles, prime and square numbers, and linear relationships. The authors note that the rigorous mathematical content of problem-based curricula combined with a demand for greater mathematical sense-making on the part of students requires teachers to adopt new lesson structures and instructional practices: engaging students in the context of the problem; activating, assessing, and utilizing students' prior knowledge; posing insightful questions that elicit students' mathematical thinking; making evident the mathematics embedded in the context of the problem; and creating connections not only to the real world but also to other areas of mathematics. The authors contend that far from being "random collections of activities" the experiences in problem-based curricula have been carefully selected and thoughtfully organized, creating coherent curricula that expose students to key mathematical concepts while developing their problem-solving skills.

Hodgson, T. (1995). Secondary mathematics modeling: Issues and challenges. *School Science and Mathematics, 96*(7), 351–358.

Mathematical modeling plays a prominent role in the mathematics reform effort. For example, modeling, as incorporated in the curriculum recommendations of the National Council of Teachers of Mathematics, forms the basis

of classroom activities developed by groups such as the Consortium for Mathematics and Its Applications (COMAP) and the Systemic Initiative for Montana Mathematics and Science (SIMMS). Modeling is also a topic of workshop and conference presentations at the state, regional, and national levels. Recommendations regarding the future of mathematics education indicate that the emphasis on modeling will continue to grow. In this article, the author presents an overview of mathematical modeling and discusses the reasons underlying the current interest in this topic. The article also identifies several practical issues that are raised by the use of modeling in secondary schools.

Isaacs, A., Wagreich, P., & Gartzman, M. (1997). The quest for integration: School mathematics and science. *American Journal of Education, 106*(1), 179–206.

Abstract: The importance of comprehensive, reform-oriented curricula for the wide-scale improvement of elementary school mathematics has long been recognized. This article is a case study of one such curriculum, *Math Trailblazers*, from the Teaching Integrated Mathematics and Science (TIMS) Project at the University of Illinois at Chicago. *Math Trailblazers* and other TIMS materials attempt to integrate mathematics and science by emphasizing science as a method and by focusing on a small set of variables that are thought to be fundamental. The TIMS materials are examined in a framework that considers the meaning of mathematics and science integration, its possible advantages and disadvantages, and the difficulties in writing and implementing integrated materials.

Martin, T., Hunt, C., Lannin, J., Leonard, W., Marshall, G., & Wares, A. (2001). How reform secondary mathematics textbooks stack up against NCTM's Principles and Standards. *Mathematics Teacher, 94*(7), 540–589. URL:<http://my.nctm.org/eresources/article_summary.asp?URI=MT2001-10-540a&from=B>.

The result of a semester-long project for graduate students in secondary school mathematics education, this article examines five comprehensive *Standards*-based high school curricula in light of NCTM's *Principles and Standards for School Mathematics.* The authors sampled material from each program and rated its coverage of NCTM's process standards and content standards. They concluded that the *Principles and Standards* were well addressed in all five curricula but that each program contained certain topics that were more strongly developed than in the other programs. The article also touches on distinctive features of each program.

Romberg, T. (1998). Designing middle school mathematics materials using problems created to help students progress from informal to formal mathematical reasoning. In L. Leutzinger (Ed.), *Mathematics in the Middle* (pp. 107–119). Reston, VA: National Council of Teachers of Mathematics. URL: <http://www.wcer.wisc.edu/ncisla/publications/articles/MiCChapter.pdf>.

Romberg offers an overview of the development of *Mathematics in Context* (MiC), a middle grades *Standards*-based curriculum, and looks at possibilities for school mathematics reform. The curriculum assumes that students will come to understand mathematics through problem-solving experiences and that the sequencing of mathematical tasks should "help students gradually develop their own methods for modeling and symbolizing problem situations." Romberg looks at algebra as a content strand developed over four years (grades 5 through 8) and describes, with illustrated examples, MiC's approach to patterns and symbols, expressions and formulas, building formulas, and patterns and figures. He shares evidence of effectiveness gathered through a review of MiC's content, information gathered from teachers, and test data. Implementation issues discussed are: sequence of the materials, coverage of topics, mathematical content, and teacher authority.

Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics, 7*(3), 154. URL: <http://investigations.terc.edu/relevant/CompFluency.html>.

Russell explains the goal of NCTM's *Principles and Standards for School Mathematics* with regard to expectations for students' computational fluency. She highlights eight main messages from *Principles and Standards* regarding computation, and defines three key ideas about fluency: efficiency, accuracy, and flexibility. She explores how fluency is grounded on strong mathematical knowledge and understanding of the following areas: meanings of operations and their relationships with one another, number relationships (including, but not limited to, "facts"), and the structure and behavior of numbers in the base-ten number system. The article includes multiple computational examples to illustrate varying levels of mathematical understanding and fluency in students, and relates some vignettes from the author's observation of elementary students' computation. She also addresses the assessment of computational fluency and provides questions to consider when analyzing students' work.

Smith, J. & Phillips, E. (2000). Listening to middle school students' algebraic thinking. *Mathematics Teaching in the Middle School, 6*(3), 156–161. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-11-156a&from=B>.

Smith and Phillips examine what algebra skills and understandings are important in K–12 mathematics learning. Throughout the article they present and discuss the work of students who are using the *Connected Mathematics* curriculum, examining the range, depth, and character of their algebraic thinking. The article emphasizes that studying student work can help teachers recognize important ideas and identify limitations in students' thinking. In their study, the authors

found that powerful algebraic ideas are accessible to students in the middle grades. Specifically, they identified five areas of knowledge in the work they examined: 1) A solid grasp of linear functions and constant rate of change; 2) The ability and flexibility to analyze functional relationships with tabular, graphical, and symbolic representations; 3) Analytic skills with graphing calculators; 4) An understanding of equivalence in each representation; 5) A beginning understanding of exponential and quadratic relationships.

Star, J., Herbel-Eisenmann, B., & Smith, J. (2000). Algebraic concepts: What's really new in new curricula? *Mathematics Teaching in the Middle School, 5*(7), 446–451. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-03-446a&from=B>.

This article examines the *Connected Mathematics* curriculum to determine "what's new" about the approach to algebra in innovative middle-school curricula. The authors note that approaches to algebra in *Standards*-based programs differ from those in traditional curricula. The newer reform-oriented programs differ with regard to: fundamental objects of study, typical problems, typical solution methods, the role of practice, the role of technology, and the elements that make up a typical lesson. The article concludes with questions about students' experiences as they progress through middle school, high school, and college; their perceptions of differences in curricular approaches; and their abilities to adapt to these different approaches.

Usiskin, Z. (1997). Applications in the secondary school mathematics curriculum: A generation of change. *American Journal of Education, 106*(1), 5–61.

Abstract: In the 1960s, the ideal curriculum as seen from recommendations in journals and reports, and the implemented curriculum as viewed from textbooks, referred very little to applications of mathematics outside the subject. Yet today the teaching of real-world applications of mathematics is seen as a necessary component of a good mathematics education. A number of factors are responsible for this change: changing enrollment trends; changing theories about how students learn and what they can learn; the arrivals of computers and calculators in schools; the public perception of performance of students on standardized tests; and the recommendations of business and industry regarding what they would like to see in the people they hire. The change is manifested in various ways beyond the inclusion of problems that relate mathematics to the world outside the classroom. The most widely used of the newer curricula develops important applications ideas from basic principles over many years. Newer influences on the thinking of mathematics educators come from advances in applied mathematics that have resulted in major changes in the workplace and a corresponding desire that no students be excluded from significant applied mathematics. As a result, some of the more recent curricula include entire courses based on units, each with a particular application theme, with the expectation that students will work both individually and in groups.

Evaluating Curricula

American Association for the Advancement of Science. (2000). *Middle grades mathematics textbooks: A benchmarks-based evaluation*. Washington, DC: American Association for the Advancement of Science. URL: <http://www.project2061.org/tools/textbook/matheval/default.htm>.

Noting that textbook analyses are "largely cursory, impressionistic, and unreliable," the American Association for the Advancement of Science (AAAS) embarked on Project 2061 in order to provide an alternative to the traditional textbook review process. Project 2061 developed a rigorous benchmarks-based tool to assess both traditional and reform-oriented middle grades mathematics curricula. Their evaluation process 1) was rigorous and uniformly applied, 2) employed an evidence-based analysis, 3) identified key mathematical benchmarks, and 4) rated each text on 24 criteria related to instruction and learning. Twelve textbook series were analyzed and rated, with the findings presented in a comprehensive report available online. Based on Project 2061's criteria the texts that received the highest rankings were: *Connected Mathematics*, *Mathematics in Context*, *MathScape*, and *MATH Thematics*. The report includes both statistical ratings and narrative summaries for each of the twelve series so that mathematics educators and textbook adoption committees can compare and contrast a wide variety of curricula and make an informed purchasing decision. Although the AAAS only reviewed curricula for the middle grades, elementary and high school committees could make their adoption process more rigorous and research-based by utilizing Project 2061's model: determining criteria, evaluating materials in light of those criteria, and creating a quantitative score and qualitative profile for each text before making a decision.

Bernhard, J., Lernhardt, M., & Miranda-Decker, R. (1999). Evaluating instructional materials. *Mathematics Teaching in the Middle School, 5*(3) 174–178. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-11-174a&from=B>.

When the authors of this article were asked to assess a set of mathematics instructional materials they began by reviewing existing criteria for evaluating curricula. After judging the existing resources to be missing key areas of focus (assessment within the materials, use of technology, and practicality of the unit), the authors developed their own criteria. They describe their process for developing an evaluation tool as well as the

rationale behind some of their decisions. The article includes their criteria for evaluating content, technology and other instructional tools, assessment, and teacher support. The authors conclude with suggestions for working with this curriculum evaluation tool.

Goldsmith, L. & Kantrov, I. (2000). Evaluating middle grades curricula for high standards of learning and performance. *National Association of Secondary School Principals (NASSP) Bulletin, 84*(615), 30–39.

This article discusses the emergence of new curriculum materials as a result of mathematics education reform, noting the challenge that educators face when selecting a curriculum. The authors describe resources designed to assist with curriculum selection and identify implementation issues that surface when using these new instructional materials. They highlight three key criteria to use when determining curricular excellence for middle grades mathematics: academic rigor, equity, and developmental appropriateness. They also discuss the characteristics of a curriculum meeting these criteria.

Grandgenett, N., Jackson, R., & Willitis, C. (2004). Evaluating a new mathematics curriculum: A district's multi-stakeholder approach. *NCSM Journal of Mathematics Education Leadership, 7*(1), 13–21.

What does an effective data-driven curriculum adoption look like? This article describes the structured curriculum evaluation and adoption process undertaken by the Westside Community Schools in Omaha, Nebraska, in collaboration with the University of Nebraska at Omaha. The district's goal was to assess the impact of a pilot implementation of *Everyday Mathematics*. The study began with a field test process involving three matched control groups and examined standardized test scores from 425 students. In addition, the district solicited direct feedback from stakeholders, using survey results from 132 teachers, 596 parents, and 2,172 students to inform the review process. This comprehensive evaluation process, designed to examine new curriculum's impact, confirmed that a full implementation of *Everyday Mathematics* was warranted.

Kulm, G. (1999). Making sure that your math curriculum meets standards. *Mathematics Teaching in the Middle School, 4*(8), 536–541. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-05-536a&from=B>.

Since new mathematics curriculum materials differ from traditional ones in appearance, content, and organization, selecting a new curriculum can be a challenging process. Kulm describes the American Association for the Advancement of Science (AAAS) analysis process that can be used to determine how rigorously curriculum materials delve into mathematical content and how effective they are at meeting standards. The analytical procedure is four-fold: selecting standards that will be used to focus the analysis, identifying components of the curricular materials that map onto these standards, assessing strengths and weaknesses of the materials through the use of over twenty specific questions (listed in the article), and creating a summary report. The author includes a straightforward explanation of differences between reform-oriented and traditional materials and concludes by briefly describing how the AAAS selection process was used by a group of middle school mathematics teachers in Kentucky. Highlights from teacher feedback on the process are also mentioned.

National Research Council. (2004). Framework for evaluating curricular effectiveness. In J. Confrey & V. Stohl (Eds.), *On Evaluating Curricular Effectiveness: Judging the Quality of K–12 Mathematics Education* (pp.36–64). Washington, DC: National Academies Press.

With the task of evaluating the effectiveness of current curriculum materials (both those supported by the NSF and those commercially generated), a committee designated by the National Research Council issued a report offering guidelines for evaluating curriculum materials and determining curricular effectiveness. This highlighted chapter lays out the framework that they created for evaluating curricula. The factors (e.g., program components (mathematical content), implementation components (professional development), student outcomes, etc.) that they articulate as determining curricular effectiveness and affecting implementation are important to consider when selecting a curriculum or reviewing the effectiveness of one's current program. The proposed framework also describes research methodologies (content analysis, comparative studies, and case studies) that can be used to study a curriculum's programmatic theory, depth, timeliness, balance, engagement, and support for diversity. The three research designs along with data from specific curriculum studies, are described in later chapters of the report. The committee also incorporates into the framework other items (e.g., evaluator independence, time elements) that need to be considered when thinking about curriculum adoption, evaluating current materials, or assessing studies involving curriculum materials.

Reys, B., Reys, R., & Chavez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61–66.

Textbooks, the second largest school expenditure after teacher salaries, are significant factors in the lives of both teachers and students. Key points made by the authors are: 1) Time pressures and prohibitive costs generally prevent textbook publishers from field-testing new materials or gathering scientific evidence to determine the effectiveness of their textbooks' content; 2) Because publishers must market material to a wide range of consumers with different adoption criteria and timelines, they are driven to make their products universally appealing and available to all states and districts; 3) Textbook authors and publishers do not necessarily base the content of their textbooks on national curriculum standards; 4) Compared to the

textbooks used in countries where students perform well on international mathematics assessments, U.S. texts address much more information and keep concepts separate rather than integrating them; 5) U.S. textbooks tend to repeat content and allow only shallow treatment of topics; 6) When it comes to choosing textbooks, "Making a wise selection is crucial because it determines the scope of mathematics that students experience and to some extent how teachers present material and how students learn." In response to these concerns, the National Science Foundation launched a major initiative resulting in K–12 textbooks that "break the mold of traditional instruction." These texts introduce skills in real-world contexts, enabling students to investigate and solve problems. The role of the teacher changes from lecturer to facilitator, and classrooms become places where mathematical questioning and conjecturing, analysis, and reflection are encouraged. The authors enumerate criteria that can help educators make sound curricular choices and advocate a strong selection team that includes teachers, administrators, and parents. Finally, the authors note that the successful introduction of *Standards*-based textbooks requires well-planned professional development to support users of the new program.

Roseman, J., Kulm, G., & Shuttleworth, S. (2001). Putting textbooks to the test. *ENC Focus, 8*(3), 56–59. URL: <http://www.enc.org/focus/literacy/document.shtm?input=FOC-002091-index>.

The American Association for the Advancement of Science's Project 2061 has conducted evaluations of 45 textbooks in middle- and high-school mathematics and science. This article highlights the problems they found in most of the textbooks reviewed, then looks more closely at some characteristics of the instructional materials they judged effective. Specifically, they draw examples from *Connected Mathematics*, a middle school mathematics curriculum, and *Matter and Molecules*, a physical science unit. These and other programs deemed effective shared four common characteristics: 1) They provided tools to help teachers take account of students' ideas coming into the lessons; 2) They engaged students with relevant contexts, experiences, and phenomena; 3) They helped students to think about and make sense of phenomena, experiences, and knowledge; 4) They helped students to develop scientific and mathematical ideas and make use of links between concepts and skills. The authors suggest some next steps for users of the Project 2061 evaluations, both in selecting curricula and in using the programs they've chosen.

St. John, M., Fuller, K. A., Houghton, N., Huntwork, D., & Tambe, P. (2000). *High school mathematics curricular decision-making: A national study of how schools and districts select and implement new curricula.* Inverness, CA: Inverness Research Associates. URL: <https://www.inverness-research.org/reports/ab_compassmonog.html>.

The research presented in this monograph explores the decision-making processes of schools and districts in choosing high school mathematics curricula, and the implications of these processes on *Standards*-based comprehensive secondary mathematics materials. The monograph reports findings from over 570 survey respondents in 1998–1999, as well as interview data from a small sub-sample of survey respondents. This monograph focuses on data about four key questions: 1) Who chooses the mathematics curriculum at the high school level? 2) What factors influence the choice of a new curriculum? 3) What is the nature of secondary mathematics curricula that are currently adopted and in use? 4) What is the level of interest in changing the high school mathematics curriculum and what is the vision for that change? Major findings reported include the fact that high school teachers play a significant role in determining curriculum; state standards have a strong influence on curriculum selection; most current high school mathematics teachers primarily rely on a traditional textbook for instruction; and most high school teachers are satisfied with their current mathematics program. The authors discuss a wide range of implications of these findings for authors of *Standards*-based curricula and their supporters and funders. Included among them are two over-arching recommendations: 1) Efforts to disseminate innovative curricula must be focused on individual teachers; 2) Dissemination of information about these curricula must help schools change their mindsets about curriculum adoption and implementation.

St. John, M., Tambe, P., Fuller, K. A., & Hirabayashi, J. (2004). *Mathematics curricular decision-making: The national landscape*. Inverness, CA: Inverness Research Associates.

This article provides an overview of factors that influence and affect curricular decision-making. Serving as an external evaluator for the K–12 Mathematics Curriculum Center, Inverness Research Associates designed a survey for K–8 mathematics curriculum leaders to document the status of curriculum in mathematics education. Respondents were representative of the national percentage of urban, rural, suburban, and small city schools but were more familiar with visions of reform. The survey questions targeted such areas as how mathematics texts are used, what factors influence the choice of mathematics curricula, and the level of familiarity districts have with reform-oriented materials. Drawing on a similar survey given to mathematics curriculum leaders at the high school level, the authors contrast the results of both surveys by sharing responses from each grade band individually and then providing an analysis of what the data mean. The findings from the survey speak to the complexity of curricular decision-making. Mathematics education leaders are often more oriented toward reform than their peers, so they face the challenge of balancing external constraints with personal and professional beliefs. The results also suggest that the process of curriculum selection and adoption are quite similar at the elementary, middle, and high school levels. When considering the status of mathematics education, the focus often turns to

looking solely at student achievement or teacher quality. Yet there are multiple factors at play, including curriculum, a factor on which schools are dependent. The authors conclude by noting that regardless of whether districts are making incremental or radical changes there is a need for ongoing curricular support.

Usiskin, Z. (1998-1999). Which curriculum is best? *UCSMP Newsletter, 24,* 3–10.

Usiskin maintains that decisions regarding which curriculum is best for students are always difficult to make. A wide variety of curricula is available, and proponents of each make a strong case for their favorite. However, data regarding the impact of these curricula are still preliminary. Additionally, decisions regarding which mathematics curriculum is best are often made or greatly influenced by individuals who have very little knowledge of the school mathematics classroom, or are made hastily by those who do not have wide curricular knowledge. To help in the review and selection process, the author provides guidelines for conducting comparative studies of different curricula.

Impact Studies

Before adopting a Standards-*based curriculum, those involved in decision-making often look for studies that provide achievement data for students using reform curricula. Acquiring students' performance data is important for many reasons, including addressing parent concerns about how these curricula affect student performance with regard to basic computation and how students using traditional vs.* Standards-*based materials compare in achievement. The studies in this section provide data about students' mathematical performance on NSF-supported curricula at the elementary, middle, and high school level. For example, several articles at the elementary level describe computational and problem-solving data for students using* Standards-*based curricula. Some of the middle school studies look at students' algebraic understanding, and one of the high school studies looks at students' preparedness for college. The research included in this section range from studies published upon the initial piloting of the curricula to those that explore the achievement of students who have had multiple years using one of the curricula.*

It is also important to note that the impact studies included here are from peer-reviewed sources—journals, conference proceedings, a book of collected articles. Other impact studies (e.g., dissertations, studies that have not been reviewed) can be accessed at each curriculum's implementation center website, which is available through http://www.edc.org/mcc/curricula.htm. Since researchers continue to explore the impact of Standards-*based curriculum on student achievement, be sure to ask about studies in progress or any recently published studies when contacting a curriculum's implementation center.*

Elementary

Baxter, J., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal, 101*(5), 529–547. URL: <http://www.aea11.k12.ia.us/sped research/res0102/research-Baxter_article.html>.

In this study, conducted over a full school year, researchers studied the classroom dynamics of reform-based mathematics instruction, paying special attention to the participation of 16 low-achieving students in five *Everyday Mathematics* classrooms. Student involvement in whole-class discussions and partnership work was observed, and their teachers were interviewed. Although the researchers saw a relatively low level of participation by these students overall, they found positive differences in classrooms where the teachers used specific strategies to increase these students' participation. These findings led the authors to conclude that in order for low-achieving students to succeed in reform-based mathematics programs, teachers and administrators must provide instructional and structural supports to encourage these students' active participation.

Briars, D. & Resnick, L. (2000). Standards, assessments—and what else? The essential elements of *Standards*-based school improvement. *CSE Technical Report 528*. Los Angeles, CA: Center for the Study of Evaluation at the National Center for Research on Evaluation, Standards, and Student Testing, UCLA. URL: <http://www.cse.ucla.edu/CRESST/Reports/TECH528.pdf>.

This paper describes the Pittsburgh Public Schools mathematics program, using data from a three-year period to explore the effects of *Everyday Mathematics* at the elementary level. The report addresses the following implementation components: content and performance standards, *Standards*-based assessment, *Standards*-based instructional materials, *Standards*-based professional development for teachers and administrators, and accountability. The authors address questions that highlight effects of *Standards*-based policy, the balance and measurement of skill mastery and conceptual understanding, achievement gains related to program implementation, curriculum, teacher quality, and the performance of minority students. Results from the analyzed studies show large gains in elementary students' mathematics learning, including improvement on norm-referenced tests that were not aligned with the curriculum. The noted improvement, however, was not uniform, which could be attributed to variability in implementation and use of the curriculum and/or variability with regard to accountability for the success of the program.

Carroll, W. (1998). Geometric knowledge of middle school students in a reform-based mathematics curriculum. *School Science and Mathematics, 98*(4), 188–197. URL: <http://www.sciencedirect.com/science?_ob=ArticleURL&_udi=B6W5B-3YMW4V6-2&_coverDate=02%2F28%2F1999&_alid=240894617&_rdoc=1&_fmt=&_orig=search&_qd=1&_cdi=6566&_sort=d&view=c&_acct=C000058919&_version=1&_urlVersion=0&_userid=2837075&md5=9b65559a32b66322f42af8aa9fc75ae9>.

Abstract: National and international studies have found U.S. elementary students to be weak in their understandings and applications of geometric concepts. The University of Chicago School Mathematics Project's (UCSMP) *Everyday Mathematics*

program is one of the current reform-based elementary curricula incorporating geometry throughout the K–6 curriculum, with an emphasis on hands-on and problem-solving activities. In this study, the geometric knowledge of fifth and sixth graders using UCSMP was compared to the knowledge of students using more traditional curricula. Because the UCSMP students had been in the program since kindergarten, this research attempts to measure the longitudinal effects of such an approach. Along with an overall score, a subset of test items was used to assign each student a van Hiele level for geometric thinking, as well as a reasoning score. On all measures, UCSMP students substantially outperformed their counterparts, and nearly all differences were significant. Aspects of the UCSMP curriculum and the van Hiele model for learning geometry are discussed relative to these results.

Carroll, W. (2000). Invented computational procedures of students in a *Standards*-based curriculum. *Journal of Mathematical Behavior, 18*(2), 111–121.

This study investigated the use of invented algorithms and computational proficiency by fourth-graders who had used the *Everyday Mathematics* curriculum since kindergarten. Through third grade, *Everyday Mathematics* encourages students to create their own computational strategies rather than emphasizing specific algorithms for addition and subtraction. For this study, students were individually interviewed and a whole-class test was administered. Researchers examined the types of student-invented procedures that students used as well as their computational accuracy. Data showed that many students created sophisticated strategies for mental calculation, while many others used the standard written algorithm, also with high accuracy. However, the study suggests that the students who used invented procedures showed greater understanding of place value, as well as better mental flexibility. Overall, the *Everyday Mathematics* students' results on the whole-class written test (fourth-grade items from the NAEP) showed their performance to be much higher than normative samples on the more challenging computation problems.

Carroll, W., & Isaacs, A. (2003). Achievement of students using the University of Chicago School Mathematics Project's *Everyday Mathematics*. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 79–108). Mahwah, NJ: Lawrence Erlbaum Associates.

The authors summarize the development of the *Everyday Mathematics* curriculum and outline eight principles that guided its development. The main focus of the chapter is a report of various studies comparing achievement of students using *Everyday Mathematics* to other groups of elementary students. Because the development of *Everyday Mathematics* began in the mid-1980s, the program has undergone a number of different kinds of studies, including: studies conducted by the University of Chicago School Mathematics Project (the developers), a longitudinal study conducted by researchers at Northwestern University, and studies done by schools and districts implementing the program. This chapter highlights three UCSMP studies: one focusing on *Everyday Mathematics* students' results on a third-grade Illinois test, one exploring students' mental computation and number sense at fifth grade, and one studying the geometric knowledge of fifth and sixth-grade students. The chapter also summarizes some results of a longitudinal study, which explored students' achievement in districts where *Everyday Mathematics* was well implemented. Lastly, it looks briefly at data independently collected by individual school districts. The authors conclude by highlighting some themes they find across the various studies, including: 1) On traditional topics like computation, *Everyday Mathematics* students perform as well as students in traditional basal programs; 2) On non-traditional elementary topics like geometry, measurement, and algebra, *Everyday Mathematics* students score substantially higher than students in traditional programs; 3) *Everyday Mathematics* students' use of "invented algorithms" offers strong evidence that the curriculum promotes students' mathematical thinking.

Carter, A., Beissinger, J. S., Cirulis, A., Gartzman, M., Kelso, C., & Wagreich, P. (2003). Student learning and achievement with *Math Trailblazers*. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 45–78). Mahwah, NJ: Lawrence Erlbaum Associates.

The research highlighted in this chapter is based upon schools in the Chicago area and nearby suburbs that had used the *Math Trailblazers* curriculum for a full two years. The authors first provide a picture of the history of the curriculum and its development, and provide excerpts from three lessons to illustrate the way the program's philosophy is embedded in the materials. The chapter then shifts to examine student outcomes with the curriculum, focusing on standardized test data as well as qualitative data. The chapter looks at *Math Trailblazers* students' performance on the Illinois Goals Assessment Program test and on the Iowa Test of Basic Skills from eight schools, comparing their performance to that of students from the same schools in years prior to the use of *Math Trailblazers*. In the case of both tests, the problem-solving approach of *Math Trailblazers* seemed to prepare students well for standardized tests. Additionally, the chapter looks at case studies of students in several different schools around the Chicago area, analyzing both their test performance and, in one case, the qualitative impact of the curriculum on students' participation in class as well as a teacher's facilitation of that class. Each of these cases showed positive impacts of *Math Trailblazers*, as well, leading the authors to conclude that "the balanced problem-solving approach found in *Math Trailblazers* has been successful in improving student learning and achievement in mathematics."

Fuson, K., Carroll, W., & Drueck, J. (2000). Achievement results for second and third graders using the *Standards*-based curriculum *Everyday Mathematics*. *Journal for Research in Mathematics Education, 31*(3), 277–295. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2000-05-277a&from=B>.

This article reports the findings of two studies that followed first-grade *Everyday Mathematics* students through second and third grade, focusing on concepts related to whole numbers and multi-digit computation. Test items used in the studies were selected either because they were considered important in new mathematics curricula, because they were from nationally administered tests such as the National Assessment of Educational Progress (NAEP), or because they were from cross-national comparisons. The findings of these two studies indicate that *Everyday Mathematics*' emphasis on problem-solving and a wider range of mathematical topics showed positive impact on the students involved in the study. At the end of the school year, *Everyday Mathematics* students in grades 2 and 3 were at normative U.S. levels for multi-digit addition and subtraction computation, and outperformed their U.S. peers in a number of specific computational areas.

Mokros, J. (2003). Learning to reason numerically: The impact of *Investigations*. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 109–131). Mahwah, NJ: Lawrence Erlbaum Associates.

The author of this chapter describes the *Investigations in Number, Data, and Space* curriculum and its major goals for mathematics teaching and learning, and then presents summaries of three studies of the curriculum's impact on students. One of the studies was conducted by TERC researchers, and two were written as doctoral dissertations. Each study poses word problems involving whole number operations, and each explores students' accuracy and effectiveness in solving problems. For each study, the background, methods, and results are outlined, and specific data are provided. The studies are:

Mokros, et al. (1994). *Full year pilot grades 3 and 4:* Investigations in Number, Data, and Space. Cambridge, MA: TERC;

Goodrow (1998). *Children's construction of number sense in traditional, constructivist, and mixed classrooms.* Doctoral dissertation, Medford, MA: Tufts University; and

Flowers (1998). *A study of proportional reasoning as it relates to the development of multiplication concepts*. Doctoral dissertation, Ann Arbor, MI: University of Michigan.

The chapter concludes with a synthesis of ideas from the three studies, including findings about students' mathematical learning and thinking in the use of *Investigations*, as well as the impact of teachers' implementation of the curriculum on students' learning.

Riordan, J. & Noyce, P. (2001). The impact of two *Standards*-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education, 32*(4), 368–398. URL: <http://www.project2061.org/meetings/textbook/policy/papers/noyce.pdf>.

Abstract: Since the passage of the Education Reform Act in 1993, Massachusetts has developed curriculum frameworks and a new statewide testing system. As school districts align curriculum and teaching practices with the frameworks, *Standards*-based mathematics programs are beginning to replace more traditional curricula. This paper presents a quasi-experimental study using matched comparison groups to investigate the impact of one elementary and one middle school *Standards*-based mathematics program in Massachusetts on student achievement. This study compares statewide standardized test scores of fourth-grade students using *Everyday Mathematics* and eighth-grade students using *Connected Mathematics* to test scores of demographically similar students using a mix of traditional curricula. Results indicate that students in schools using either of these *Standards*-based programs as their primary mathematics curriculum performed significantly better on the 1999 statewide mathematics test than did students in traditional programs attending matched comparison schools. With minor exceptions, differences in favor of the *Standards*-based programs remained consistent across mathematical strands, question types, and student sub-populations.

Sconiers, S., McBride, J., Isaacs, A., Kelso, C., & Higgins, T. (2003). *The ARC Center Tri-state Achievement Study*. Lexington, MA: COMAP. URL: <http://www.comap.com/elementary/projects/arc/tri-state%20achievement%20full%20report.htm>.

In this study funded by the National Science Foundation (NSF) the ARC Center looked at the effects of three elementary curricula designed to align with the vision of the NCTM *Standards*. The *Tri-state Achievement Study* compares the achievement of elementary students using *Everyday Mathematics*, *Investigations*, and *Math Trailblazers* to students not using these materials. In this large-scale study, the ARC Center gathered math achievement data, state mandated test scores, and survey data from schools in Illinois, Massachusetts, and Washington, all states with large numbers of students using one of the three NSF programs. Students needed to have had at least two years of exposure to the program in order to be included in the data. A set of comparison schools that were not using any of the three reform curricula were then matched according to reading score, socioeconomic level and other factors, including percent of white and Title I students. The authors examined achievement comparisons at multiple levels (e.g., family income, race) and consistently found that when a statistical difference in math scores was detected for a particular math strand (e.g., geometry, algebra) the students using one

of the three NSF curricula were always favored. In general, the authors found that the reform curricula yielded an improvement in student performance on many levels including basic skills and higher-order processes.

Middle School

Bay, J., Beem, J., Reys, R., Papick, I., & Barnes, D. (1999). Student reactions to *Standards*-based mathematics curricula: The interplay between curriculum, teachers, and students. *School Science and Mathematics, 99*(4), 182–188.

Abstract: As *Standards*-based mathematics curricula are used to guide learning, it is important to capture not just data on achievement but data on the way in which students respond to and interact in a *Standards*-based instructional setting. In this study, sixth and seventh graders reacted through letters to using one of two *Standards*-based curriculum programs, *Connected Mathematics* (CMP) or Six Through Eight Mathematics (*MATH Thematics*). Letters were analyzed by class, by teacher, and by curriculum project. Findings suggest that across classrooms students were positive toward applications, hands-on activities, and working collaboratively. The level of students' enthusiasm for the new curricula varied greatly from class to class, further documenting the critical role teachers play in influencing students' perceptions of their mathematics learning experiences. The results illustrate that, while these curricula contain rich materials and hold much promise, especially in terms of their activities and applications, their success with students is dependent on the teacher.

Ben-Chaim, D., Fey, J., Fitzgerald, W., Benedetto, C., & Miller, J. (1997). *A study of proportional reasoning among seventh and eighth grade students*. Paper presented at the Annual Meeting of American Education Research Association, Chicago, IL. URL: <http://www.math.msu.edu /cmp/RREvaluation/Support/BenChaim.htm>.

This paper reports a study of students' performance on proportional reasoning tasks presented in various contexts. Over all types of contexts, students using *Connected Mathematics* (CMP) performed approximately fifty percent better than students in non-CMP classes. In addition, the study suggests that seventh-grade students in CMP increased their proportional reasoning abilities by the end of eighth grade, without any further formal study of proportional reasoning.

Billstein, R. & Williamson, J. (2003). Middle Grades *MATH Thematics:* The STEM Project. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 251–284). Mahwah, NJ: Lawrence Erlbaum Associates.

The authors open this chapter with an explanation of the *MATH Thematics* curriculum, highlighting its goals for students. They describe the mathematical content strands covered in the curriculum and point out unifying concepts that extend throughout the curriculum such as proportional reasoning, multiple representations, patterns and generalizations, and modeling. They also explain assessment tools that are integral to the curriculum. The chapter shifts to look at evidence of student achievement in *MATH Thematics*. Since the program only became available to schools in 1998, data on its impact are limited. However, the authors report research from field-test versions of the curriculum, from outside resources, and from formative evaluations conducted during the curriculum's development that seem to indicate positive effects. While the data does not show a significant effect on measures of traditional mathematics achievement, there has been significant improvement in students' reasoning, communication, and mathematics problem-solving abilities as well as a more positive attitude toward mathematics. Additionally, preliminary data suggest that the curriculum may contribute positively to improved achievement in reading and language arts. The chapter concludes by underscoring that teachers need resources and thorough preparation to use these materials effectively, and that additional studies need to be conducted in order to verify field-test results.

Krebs, A. (2003). Middle grades students' algebraic understanding in a reform curriculum. *School Science and Mathematics, 103*(5), 233–243.

"Algebra for All" is often implemented by teaching younger and younger students symbol manipulation in traditional ways. Are there curricular alternatives to this approach? In a *Standards*-based program, where algebra is "woven through the curriculum rather than being parceled into a single grade level," is the content rigorous enough to meet even the needs of high-achieving students? Krebs studied the ability of middle- to high-achieving students in their third year of using *Connected Mathematics* (CMP) to create symbolic expressions for performance tasks involving linear, quadratic, and exponential functions. The author found that students could demonstrate proficiency in five significant strands of algebra, and that in 24 of 29 cases they were able to write correct symbolic expressions for a given task. (In the remaining five cases, even students who had not yet been exposed to the task's content were still able to demonstrate partial understanding.) Krebs discovered that strategies learned in their *Standards*-based program provided these students with an entry into unfamiliar problems, and while they "might not use standard algorithms to find their expressions, they used their understanding to determine the correct generalization." In addition when interviewed by the author all students showed the ability to justify their work, and when they were confronted with a challenging problem containing new content, they persevered. Most importantly, students showed a deep understanding of

the mathematics by making connections between representations, and between linear, quadratic, and exponential functions. The article includes the actual performance tasks, questions used to probe student thinking, samples of students' written work, and excerpts from student interviews. These features allow the reader to experience what it is like for a teacher using a *Standards*-based program to make sense of unanticipated student responses, varied representations, and symbolic expressions that although equivalent, initially look very different.

Miller, J. & Fey, J. (2000). Proportional reasoning. *Mathematics Teaching in the Middle School, 5*(2), 310–313. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-01-310a&from=B>.

This article explores student responses from an earlier study of proportional reasoning among seventh-grade students. In that study, (available online at http://www.math.msu.edu/cmp/RREvaluation/Support/BenChaim.htm), researchers compared responses on several proportional reasoning tasks of students who were studying in a *Standards*-based middle-grades mathematics curriculum (*Connected Mathematics*) with the responses of students in a control group who were using traditional curricula. The article reports that students using *Standards*-based materials were more successful both in giving correct responses to the tasks and in explaining their reasoning. While noting that middle school students, in general, have considerable room for growth in the understanding of proportional reasoning, the authors concluded from their research that curriculum and instruction aligned with the NCTM *Standards* can help students construct understanding and skill in this core strand of middle-grades mathematics.

Reys, R., Reys, B., Barnes, D., Beem, J., Lapan, R., & Papick, I. (1998). *Standards*-based middle school mathematics curricula: What do students think? In L. Leutzinger (Ed.), *Mathematics in the Middle* (pp. 153–157). Reston, VA: National Council of Teachers of Mathematics.

This article reports on student response to four NSF-funded *Standards*-based middle-school mathematics curricula (*MATH Thematics*, *Connected Mathematics*, *Mathematics in Context*, and *MathScape*) used in 15 midwestern school districts. The responses students gave about what mathematical experiences they liked or disliked was often unexpected, and fell into five categories: 1) Is this mathematics? 2) Doing mathematics is better than hearing about it; 3) Solving problems is hard/challenging; 4) The "old" mathematics was more comfortable; 5) Working in groups is good/hard/awful. Sample responses included in the article are: "It wasn't really like math"; "I liked this material because it had things related to life"; "I didn't really like the books because it was a lot of reading"; "It makes learning easier because you get to actually do it"; and "Solving problems is very complicated. It seems to me that there are simpler ways of doing math." In conclusion, the authors recognize the importance of the teacher's role in impressions students have about mathematics and the learning process.

Reys, R., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of *Standards*-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34*(1), 74–95. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2003-01-74a>.

This article describes differences in achievement between eighth graders who had used one of the NSF-funded curricula *MATH Thematics or Connected Mathematics Project* (CMP) for at least two years and students who had used traditional programs throughout their middle school experience. After matching three pairs of districts for size, demographics, and similar achievement in mathematics prior to the introduction of reform curricula, student scores on both the statewide eighth grade assessment and *Terra Nova*, a nationally norm-referenced assessment, were compared. The authors found that the students using reform curricula "equaled or exceeded the achievement of students from the matched comparison districts." Further, whenever a higher significant difference occurred, it "reflected higher performance for students using NSF *Standards*-based materials." They also note, "Although critics have chastised NSF *Standards*-based curricula for ignoring basic skills, this research does not support that claim," and that contrary to the complaint that these curricula do not give enough attention to algebra, "significant differences occurred across all three (reform curricula) groups on the algebra portion" of the state assessment.

Ridgway, J., Zawojewski, J., Hoover, M., & Lambdin, D. (2003). Student attainment in the *Connected Mathematics* curriculum. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 193–224). Mahwah, NJ: Lawrence Erlbaum Associates.

This chapter explains the design and instructional philosophy of the *Connected Mathematics* (CMP) curriculum, highlighting its organization around problem settings and its focus on mathematical concepts and applications. The chapter shares findings from three different studies of CMP's impact on students in the program. The first study compares the performance of CMP and similar non-CMP students on the Iowa Test of Basic Skills (ITBS) test and the Balanced Assessment test at grades 6, 7, and 8. In addition to looking at comparison data from schools nationwide, the chapter also explores one rural district's data in more depth. The second study examines the changes in seventh-grade CMP students' performance on the Michigan Educational Assessment Program (MEAP) annually from 1992 to 1999. The third study features results of nationally-sampled CMP and non-CMP seventh grade students on a testing instrument featuring

proportional reasoning problem situations, for which student work was coded for correct responses as well as for correct support to their responses. Overall findings of these studies include: 1) CMP students made large gains on a broad range of topics and processes, and reasonable gains on technical skills, as compared to non-CMP students; 2) CMP students showed evidence of long-term gains when CMP was the sole curriculum for all of the middle grades. Questions for further research include the impact of the design and quality of implementation of the curriculum, and the effects of the curriculum on different student populations.

Riordan, J. & Noyce, P. (2001). The impact of two *Standards*-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education, 32*(4), 368–398. URL: <http://www.project2061.org/meetings/textbook/policy/papers/noyce.pdf>.

Abstract: Since the passage of the Education Reform Act in 1993, Massachusetts has developed curriculum frameworks and a new statewide testing system. As school districts align curriculum and teaching practices with the frameworks, *Standards*-based mathematics programs are beginning to replace more traditional curricula. This paper presents a quasi-experimental study using matched comparison groups to investigate the impact of one elementary and one middle school *Standards*-based mathematics program in Massachusetts on student achievement. This study compares statewide standardized test scores of fourth-grade students using *Everyday Mathematics* and eighth-grade students using *Connected Mathematics* to test scores of demographically similar students using a mix of traditional curricula. Results indicate that students in schools using either of these *Standards*-based programs as their primary mathematics curriculum performed significantly better on the 1999 statewide mathematics test than did students in traditional programs attending matched comparison schools. With minor exceptions, differences in favor of the *Standards*-based programs remained consistent across mathematical strands, question types, and student subpopulations.

Romberg, T. & Shafer, M. (2003). *Mathematics in Context* (MiC): Preliminary evidence about student outcomes. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 225–250). Mahwah, NJ: Lawrence Erlbaum Associates.

The authors explain that the *Mathematics in Context* (MiC) curriculum is the result of a collaboration between the research and development teams at the Wisconsin Center for Education Research at the University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht in the Netherlands. The curriculum was designed to develop four mathematical strands (number, algebra, geometry, and data and statistics) conceptually across units and grade levels. The approach was based upon a Dutch model called "Realistic Mathematics Education" that showed positive effects on student learning in the Netherlands. The chapter also discusses the various types of research conducted during the development of the curriculum, as well as research in progress about the curriculum's impact on students. Primarily, the authors explain that data on student achievement were collected from pilot tests, field tests, external evidence, and case studies, and that a longitudinal/cross-sectional study was in progress. Pilot test data were used to revise early versions of the curriculum and did not provide common student outcome data. Field tests generated student achievement data for specific units; while demonstrating variation in classroom testing, the data did show evidence that use of the curriculum elicited high achievement by students at all ability levels. District-collected (external) student performance data showed positive student outcomes on standardized tests like the ITBS, NSRE, and others. Case study data highlighted the qualitative impact of MiC on teachers and students and also showed student progress and enthusiasm about the program. Finally, the authors reflect upon the "complex social context" that influences the longitudinal study of the program.

Shafer, M. C. (2003). *The impact of* Mathematics in Context *on student performance*. Paper presented at the Research Presession of the National Council of Teachers of Mathematics, San Antonio, TX.

This research used pairs of matched districts to study the impact of *Mathematics in Context* (MiC) by examining student achievement with regard to variables related to instruction, opportunity to learn with understanding, gains in relation to prior achievement, and measures of school capacity. Progress maps displaying several types of comparisons (grade-level-by-year, cross-grade studies, and cross-year studies) are included to show six levels or bands of student achievement. Preliminary results of the cross-sectional studies have indicated that MiC has had positive results on student performance when used over time, and that in the cross-year comparisons, "the performance of students studying MiC was significantly higher than performances of students in previous years in three of the four research sites." In order to learn more about potential factors that may have affected students' mathematical performance the research team also sought to "capture the essence of the culture in which learning is situated," looking for "commonalities and differences among teachers." A continuum featuring four categories of teacher behavior and classroom environment was developed to assess opportunities to learn with understanding. In order to highlight differences in classroom culture, Shafer offers the reader a window into the mathematics classrooms of two participating teachers and then raises the question of to what extent teaching practices have a positive or adverse effect on students' performance.

Show-Me Center. (2001). Research on use of *Standards*-based middle-grades curriculum materials. *Show-Me Center Brief*. Columbia, MO: Show-Me Project. URL: <http://showmecenter.missouri.edu/resources/ResearchBrief.pdf>.

This article provides a summary of research on the use of *Standards*-based mathematics curricula in the middle grades. In addition to noting student achievement studies, the article also highlights research on the impact new curricula have had on teachers' practice and their use of the new materials. It closes with a summary of research findings, indicating important points about teachers' needs when implementing *Standards*-based curricula, as well as students' responses to using these programs.

Zawojewski, J., Robinson, M. & Hoover, M. (1999). Reflections on developing formal mathematics and the *Connected Mathematics Project. Mathematics Teaching in the Middle School, 4*(5), 324–330. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-02-324a&from=B>.

The authors of this article examined the work of sixth through eigth grade *Connected Mathematics* students on a task dealing with area, and observed clear growth from sixth to eigth grade in mathematical representation, use of formulas, applying procedures, and mathematical communication. Specifically, the study found that students had developed facility in later grades with concepts that were emphasized in sixth grade, but not formally retaught later. Students were observed working on problems requiring them to use what they knew (such as representing properties of plane geometric figures, or applying proportional reasoning and number skills) with increasing degrees of sophistication. The authors believe that it was through application that eigth grade students had developed a more complex understanding of area concepts and greater facility with them. The article concludes with questions to prompt further thinking about the development of students' conceptual understanding and the implications for mathematics instruction.

High School

Cichon, D., & Ellis, J. (2003). The effects of *MATH Connections* on student achievement, confidence, and perception. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 345–374). Mahwah, NJ: Lawrence Erlbaum Associates.

The authors frame this paper by highlighting the goals of *MATH Connections: A Secondary Mathematics Core Curriculum* and their influence on the design of the materials. *MATH Connections* is designed to connect education, students, and business through mathematics; to make students powerful mathematics learners; and to help teachers enact the NCTM *Standards* in their classrooms. The evaluation of the curriculum is described as being "multi-faceted," including data from students' achievement on standardized tests, a scale rating students' attitudes and perceptions, classroom observation data, data from curriculum quizzes, and case studies in three districts. The design of this evaluation is meant to address 1) how students perform on independent standardized tests; 2) whether participating students achieve the objectives of the curriculum; 3) how achievement in the curriculum is related to students' gender, ethnicity, and special education levels; 4) how *MATH Connections* students' attitudes toward mathematics learning differ from students not in the program; and 5) what *MATH Connections* classrooms look like. In general, the evaluation data show that *MATH Connections* students perform as well or better than their peers on standardized tests, and the program accomplishes its goals with students. Students in classrooms where *MATH Connections* is taught and whose teachers have been trained in the program demonstrate higher confidence and more positive attitudes about mathematics than do their peers in non-*MATH Connections* classrooms.

Huntley, M., Rasmussen, C., Villarubi, R., Sangtong, J., & Fey, J. (2000). Effects of *Standards*-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand. *Journal for Research in Mathematics Education, 31*(3), 328–361. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2000-05-328a&from=B>.

Abstract: Students in *Contemporary Mathematics in Context* (CPMP) course 3 and those in more traditional Algebra II classes, matched on measures of eighth-grade mathematics achievement, were administered a researcher-developed test of algebraic understanding, problem-solving and procedural skill at the end of the school year. CPMP students scored

significantly better on the subtests of understanding and problem solving, and Algebra II students scored significantly better on the subtest of paper-and-pencil procedures. Scores and student work are discussed by item in this paper.

Kramer, S. (2003). *The joint impact of block scheduling and an NCTM Standards-based curriculum on high school mathematics achievement*. College Park, MD: University of Maryland. URL: <http://lsc-net.terc.edu/media/data/media_000000000753.pdf>.

Considering a change to block scheduling inevitably raises concerns for parents and educators alike. Those concerns can be compounded when block scheduling meets another innovation: the adoption of a *Standards*-based curriculum. Kramer's research findings, however, should be reassuring to stakeholders. Although block scheduling alone, without change in curriculum and instruction, may result in lower mathematics achievement, "when a semestered block schedule and the *Interactive Mathematics Program* (IMP) curriculum were implemented jointly, with extra time allocated to planning and staff development, the two innovations were followed by improvements in students' mathematics achievement." The author shares three other findings that may be of interest to parents and stakeholders: 1) Students in block scheduled IMP classes took more hours of mathematics courses; 2) Because sizeable numbers of students completed the four IMP courses, many had time to take between one and five advanced mathematics courses before finishing high school; 3) When Honors students taking IMP were compared to Honors students taking the traditional algebra I, geometry, algebra II curriculum under block scheduling, "the IMP groups scored significantly and substantively higher on NAEP Conceptual Understanding items than traditionally taught students."

Lott, J., Hirstein, J., Allinger, G., Walen, S., Burke, M., Lundin, M., et al. (2003). Curriculum and assessment in *SIMMS Integrated Mathematics*. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 399–424). Mahwah, NJ: Lawrence Erlbaum Associates.

The authors of this chapter provide a history of the development of *SIMMS Integrated Mathematics*. The curriculum was intended to be 1) integrated and interdisciplinary, 2) problem-centered and applications-based, 3) appropriately useful of technology as a learning and teaching tool, 4) sensitive to multiple perspectives and stereotypes, and 5) accommodating to multiple learning styles. The chapter provides a look at the assessment principles that guided the program's development. Then the authors summarize the range of research about students' achievement with the curriculum, including pre-pilot and pilot studies in Montana, pilot studies in an urban district outside of Montana, and a study of Montana college freshmen who had taken three years of the SIMMS curriculum. The pre-pilot studies primarily served as formative research to inform both revisions of the curriculum and methods for evaluating student outcomes. The pilot studies conducted in Montana compared SIMMS classes' and control classes' outcomes on the PSAT and an end-of-year task assessment. The data showed no significant differences on PSAT scores and showed that SIMMS students were more successful on end-of-year tasks, especially during the earlier years of the program (i.e., grades 9 and 10). The pilot studies in the urban district showed SIMMS students at least matching the performance of Algebra I students on the PSAT and scoring slightly better on open-response problems. Finally, in the Montana college freshmen study, the majority of SIMMS students responded either positively or neutrally when asked if SIMMS had helped to prepare them for their first college mathematics course.

Resek, D. (1999). *Evaluation of the* Interactive Mathematics Program. Paper presented at the Annual Meeting of the American Educational Research Association, Montreal, Canada. URL: <http://www.mathimp.org/research/AERA_paper.html>.

Resek presents an overview of the *Interactive Mathematics Program* (IMP), a four-year secondary curriculum, and summarizes the extensive IMP evaluation performed by the Wisconsin Center for Education Research (WCER). Evaluation results are grouped and studies are discussed in the following categories: high school grades and retention, student performance on standardized tests, performance comparison on other tests, comparison of attitudes, and comparison of performance after secondary school. The article concludes with research questions that address how to find a good control group after high school, whether positive effects of a curriculum lessen as practitioners are distanced from the developers, and how the role of the teacher can be accounted for in evaluation data. This article's references include seven studies that evaluate the IMP curriculum:

Clarke, D., Wallbridge, M. & Fraser, S. (1996). *The other consequences of a problem-based mathematics curriculum*. Research Report No. 3. Victoria, Australia: Mathematics Teaching and Learning Centre.

Webb, N. & Dowling, M. (1996). *Impact of the Interactive Mathematics Program on the retention of students: Cross-school analysis of transcripts for the class of 1993 for three high schools*. Madison, WI: Wisconsin Center for Education Research.

Webb, N. & Dowling, M. (1997). *Comparison of IMP students with students enrolled in traditional courses on probability, statistics, problem solving, and reasoning*. Madison, WI: Wisconsin Center for Education Research.

Webb, N. & Dowling, M. (1998). *Replication study of the comparison of IMP students with students enrolled in traditional courses on probability, statistics, problem solving, and reasoning*. Madison, WI: Wisconsin Center for Education Research.

White, P., Gamoran, A., & Smithson, J. (1995). *Math innovations and student achievement in seven high schools in California and New York*. Madison, WI: Consortium for Policy Research in Education and the Wisconsin Center for Education Research.

Wolff, E. (1997). *Summary of matched-sample analysis comparing IMP and traditional students at Philadelphia High School for Girls on mathematics portion of Stanford-9 test*. Glenside, PA; Mathematics Department, Beaver College.

Wolff, E. (1997) *Summary of matched-sample Stanford-9 analysis comparing IMP and traditional students at Central High School*. Philadelphia, PA: Mathematics Department, Beaver College.

Schoen, H., Cebulla, K., Finn, K., & Fi, C. (2003). Teacher variables that relate to student achievement when using a *Standards*-based curriculum. *Journal for Research in Mathematics Education, 34*(3), 228–259. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2003-05-228a&from=B>.

This paper details a study of 40 teachers using the *Contemporary Mathematics in Context* (Core-Plus) curriculum with 1,466 students in 26 schools, with the goal of identifying specific teacher behaviors that positively affect the performance of students using a reform mathematics curriculum. Although previous research has shown that *Standards*-based curricula can have a positive effect on student understanding and success in mathematics, this study looked closely at exactly which teacher practices correlated with student achievement. To do this, researchers examined teachers' preparation, practice, and concerns as they used Core-Plus. They found that one of the best predictors of student achievement was whether or not a teacher attended a two-week summer workshop in preparation for teaching Course 1. Another factor that seemed to be positively associated with achievement was one of management: teachers in this category felt capable of managing the changes involved in implementing the new math program. Observations of teachers in the classroom led to the conclusion that teaching strategies that were consistent with the NCTM *Standards* and with Core-Plus guidelines yielded greater student achievement. In addition, a positive association was also found between high student achievement and the teachers' "high academic expectations on homework, grading and maintaining the integrity of the curriculum materials and assessments." These results were shown to be true for a range of students coming from different backgrounds and placed in different school situations. The findings of this study show a strong relationship between high student achievement and teaching a *Standards*-based mathematics curriculum using the instructional practices recommended by the authors and developers. In other words, if the curriculum is taught the way it was intended, students are more likely to reap a reform curriculum's benefits. These results underscore the importance of aligning teacher practice with the goals of the curriculum in order to promote student understanding and success in mathematics. They also suggest that differing levels in the quality of an implementation will yield differing levels of success in student achievement.

Schoen, H. & Hirsch, C. (2003). Responding to calls for change in high school mathematics: Implications for collegiate mathematics. *American Mathematical Monthly, 110*(2), 109–123. URL: <http://www.wmich.edu/cpmp/pdfs/monthly109-123.pdf>.

In some districts, especially those where high-achievement in mathematics has been the norm, the use of reform curricula such as *Contemporary Mathematics in Context* (Core-Plus) has become a focal point for debate. Schoen and Hirsch address this issue by posing two questions: How does the pattern of student learning in Core-Plus differ from that of comparable students using traditional curricula? Do reform curricula prepare students for college mathematics? After describing features that distinguish a reform curriculum from a traditional one, the authors cite statistics comparing the performance of Core-Plus and traditionally taught students on several widely used tests, including Educational Testing Service's *Algebra End of Course Evaluation* and *SAT I Mathematics*. Their findings show that while there was a great deal of overlap, Core-Plus students almost always performed better than students in the comparison groups on measures of higher-order thinking in mathematics such as conceptual understanding, interpretation of mathematical representations and calculations, and problem-solving in applied contexts. (Paper-and-pencil equation solving, a skill often practiced extensively by traditionally taught students, was the one area where Core-Plus students underperformed the comparison groups. Material to address this was added to subsequent editions of Core-Plus.) With regard to readiness for college mathematics, the authors found that Core-Plus students performed at a higher level than pre-calculus students on the concepts and applications sections of placement tests. In addition, when studying data regarding course enrollment and grades in Calculus I and higher mathematics courses they found that Core-Plus "did no harm to either the percent of students enrolling in these courses or their course grades—and may have helped the latter." This article would also be useful in informing college mathematics faculty about changes in curriculum at the secondary school level. The authors note that although undergraduate mathematics and reform secondary curricula both include similar topics (e.g., probability and statistics, discrete mathematics, matrices and mathematical modeling) traditional placement tests seldom assess this material. They suggest that the content of these assessments should be evaluated and updated so that students who are well prepared for the rigors of undergraduate mathematics are not penalized by tests with an undue emphasis on symbol manipulation.

Schoen, H. & Hirsch, C. (2003). The *Core-Plus Mathematics Project:* Perspectives and student achievement. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 311–344). Mahwah, NJ: Lawrence Erlbaum Associates.

This chapter provides an overview of the *Contemporary Mathematics in Context* (Core-Plus) curriculum's design and theoretical framework, as well as its goals for student learning. In addition, the authors present a profile of student outcomes in their use of the curriculum. Achievement results are reported from the three-year Core-Plus field test (1994-97) on the standardized Ability to Do Quantitative Thinking (ATDQT) test. Results on measures of students' understanding of algebraic and geometric concepts and methods, and of statistics, probability and discrete mathematics are also presented. Students' perceptions and attitudes about mathematics and about their mathematics course are summarized. SAT and ACT scores of students using Core-Plus are compared to those using more traditional curricula. Lastly, data are reported from a study of grades in college mathematics classes for students who used Core-Plus materials and those who took other courses in high school. On all measures except paper-and-pencil algebra skills, students who used Core-Plus did as well as or better than those who used traditional curricula.

Webb, N. (2003). The impact of the *Interactive Mathematics Program* on student learning. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 375–398). Mahwah, NJ: Lawrence Erlbaum Associates.

After providing an explanation of the *Interactive Mathematics Program* (IMP) and the philosophy behind its design, this chapter discusses a range of evaluation studies of the IMP curriculum. Because the curriculum incorporates traditional mathematics topics (algebra, geometry, and trigonometry), as well as topics that historically have not been stressed in high school mathematics, such as statistics and probability, the evaluations were designed to examine students' achievement in both traditional and newer topic areas. The first evaluation study reported by the author was a transcript analysis across three high schools, focusing on students' grades, mathematics courses taken, and students' test scores, comparing IMP students with students in traditional college preparatory courses. The study showed that a slightly higher percentage of students completed three years of IMP than completed three years of a more traditional college-preparatory sequence, and that 26% more IMP students went on to take more advanced mathematics courses. The transcript study also showed that IMP students and students in traditional courses scored comparably on common mathematics achievement tests like the CTBS and the SAT. The second set of evaluation studies analyzed IMP students' performance on tests of mathematical content emphasized by the curriculum, primarily statistics, probability, and complex problem-solving. The study was conducted in 1995-1996 and then replicated in different schools in 1996–1997. Findings included: ninth grade IMP (Year 1) students scored significantly higher on statistics items than did students who were enrolled in Algebra I; tenth grade (Year 2) IMP students outperformed traditional students on two performance assessments measuring students' reasoning, problem-solving, and application skills; IMP Year 3 students (eleventh graders) scored significantly higher than Algebra II students on a range of topics, including normal curve properties, extrapolation using rates and percentages, and probability.

Professional Development

Standards-*based curricula place new demands on teachers. There is new content to teach, such as statistics and discrete mathematics. Research-based pedagogy incorporated into curricular design requires teachers to press for discourse and communication in their mathematics classrooms. Effective management of group work, use of multiple forms of assessment, and analysis of unfamiliar solution strategies are also expected. Most importantly, student mathematical understanding must stay sharply in focus as the central goal of teaching. Since most teachers, both classroom veterans and those in pre-service programs, have not experienced this type of mathematics education, professional development is crucial if they are to make a shift to a content-rich, problem-based, student-centered model of teaching and learning.*

The articles in this section focus on providing professional development in the mathematical and pedagogical content knowledge that teachers need to know in order to teach Standards-*based curricula in a* Standards-*based manner. Also included are articles that share models of classroom instruction and give voice to teachers who have already adopted* Standards-*based practices. Finally, if classroom practice is to change, so must models of observation, supervision, and evaluation. To this end, articles that inform and support the professional development of administrators are also included.*

Teacher Learning

Ball, D. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77*(7), 500–508.

Deborah Ball writes about the movement to adopt NCTM *Standards*-based curricula, and of "scaling up" teacher training. Ball reviews nine main ideas that reflect current understandings of teacher learning and *Standards*-based teaching: prior beliefs and experience, subject matter knowledge, hearing and developing knowledge of students, the importance of the contexts in which teachers teach, time, reflection, follow-up, modeling, and teacher control. Challenges discussed in this article are: incomplete knowledge, competing commitments, and anticipating, interpreting, and responding to students. Ball reminds readers of the paradox that today's teachers are "the products of the very system they are now trying to reform," and addresses the complexities of introducing new teaching and learning methods to seasoned teachers. She also addresses differences between pre-service and in-service teacher development. Many of Ball's beliefs about learning center on open questioning (posed to students and teachers) and the use of curriculum materials as a means for inquiry into the field of teacher learning. Ball also writes about the use of video as a tool to help teachers visualize different teaching methods and student reactions to them.

Ball, D. (2003). *What mathematics knowledge is needed for teaching mathematics*? Presentation at the Secretary's Summit on Mathematics of the U.S. Department of Education, Washington, DC.

In this paper, Ball covers some of the same ground that many others concerned with mathematics education have gone over before, but then takes it a step further into a deeper discussion of the substantial work that is involved in being a skilled teacher of mathematics. Although encouraging teachers to increase their knowledge of mathematics is necessary, it is not sufficient. Ball says, "[I]ncreasing the quantity of teachers' mathematics coursework will only improve the quality of mathematics teaching if teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work. The goal is not to produce teachers who know more mathematics. The goal is to improve students' learning. Teachers' opportunities to learn must equip them with the mathematical knowledge and skill that will enable them to teach mathematics effectively." Ball goes on to discuss what mathematical knowledge is needed to be an effective instructor and makes a distinction between knowing how to do math and knowing how to teach math to students. Teaching, she says, "requires significant mathematical skill, insight, and understanding, again well beyond the knowledge required to carry out a procedure oneself." Ball includes examples of mathematics problems that help to illustrate her message. She unpacks the work of teaching mathematics and leaves the reader with a much clearer idea of the complexity and seriousness of a profession that has seen some improvement over the last decade, but needs much, much more. "The improvement of mathematics teaching in this country depends on, among other things, the improvement of our understanding of its mathematical nature and demands, and the provision of opportunities for professionals to acquire the appropriate mathematical knowledge and skill to do that work well."

Cwikla, J. (2004). Show me the evidence: Mathematics professional development for elementary teachers. *Teaching Children Mathematics, 10*(6), 321–326. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=TCM2004-02-321a&from=B>.

In recent years, the field of mathematics education has seen much research done on teacher development, teacher change, and teacher professional development. This article reviews some of the empirical research that has been conducted at the elementary level with the goal of providing useful guidelines for the design of mathematics professional development for empirical research that has been conducted at the elementary level with the goal of providing useful guidelines for the design of mathematics professional development for elementary school teachers. The author has identified four key characteristics of effective professional development: 1) Focus teachers' thinking and learning on *students*' thinking and learning; 2) Foster a collegial environment in which teachers believe they can learn from one another; 3) Offer teachers sustained rather than short-term professional development to help them both understand new ideas and give them time to change their practice; 4) Provide opportunities for teachers to test their theories in their classrooms in order to better understand the impact of their teaching on student learning.

Davenport, L. (2000). *Elementary mathematics curricula as a tool for mathematics education reform: Challenges of implementation and implications for professional development*. Center for the Development of Teaching (CDT) Paper Series. Newton, MA: Education Development Center, Inc.

Abstract: A number of *Standards*-based elementary mathematics curricula have been created to serve as a tool for mathematics education reform. Although these curricula have much to offer teachers, they also pose serious challenges, and in order to use these curricula as intended, teachers must shift how they think about mathematics, mathematics learning, and mathematics teaching. This paper provides two stories of teachers learning to work with an innovative elementary mathematics curricula, while they are participating in a year-long *Developing Mathematical Ideas* seminar. In the first story, a teacher using *Investigations in Number, Data, and Space* is working through the question of what her students should be learning, and as she learns more mathematics herself, she finds that she is better able to articulate mathematics learning goals for her students. In the second story, a teacher using the *Everyday Mathematics* curriculum is developing a curiosity about her students' mathematical thinking, and as she becomes more intrigued with the different ways her own students are thinking about the problems she is posing, she begins to make more space for their thinking in her classroom. An examination of these stories shows how professional development that engages teachers in thinking deeply about the mathematics content of the elementary mathematics curriculum and exploring how students think about that mathematics content can help prepare teachers to use *Standards*-based curricula as a tool for reforming their practice.

Empson, S. & Junk, D. (2004). Teachers' knowledge of children's mathematics after implementing a student-centered curriculum. *Journal of Mathematics Teacher Education, 7*(2), 121–144.

Empson and Junk describe how teaching the *Investigations* curriculum provided an opportunity for teachers to learn about children's mathematics. The authors define children's mathematics as a teacher's knowledge of concepts, procedures, and mathematical practices that are integrated with a knowledge of children's thinking. The authors interviewed 13 elementary school teachers in their first or second year of using *Investigations* with the goals of understanding how teachers made sense of students' nonstandard strategies for multi-digit operations and how teachers' increased knowledge was linked to the use of a new innovative curriculum. The five-question interview included open-ended questions and scenarios specifically related to teachers' learning from *Investigations* and their knowledge and understanding of nonstandard strategies for multi-digit operations. For this group of teachers, the quality of responses appeared related to the "opportunity for teacher learning created by the curriculum." Teacher responses demonstrated deeper knowledge of students' nonstandard strategies for multiplication rather than for division or subtraction, a result that was not surprising given that multiplication was more extensively developed in the curriculum. Empson and Junk found that the curriculum showed teachers that students could generate strategies and solve problems and this, in turn, motivated them to extend their own understanding. The authors found that as a direct result of teaching *Investigations* teachers developed new beliefs about mathematics in which they valued nonstandard strategies and recognized their students' ability to generate mathematical ideas and solve problems on their own. Teachers' developing beliefs reinforced the importance of basing instruction on children's mathematics and provided a context for their own learning. The teaching of *Investigations* both served as an entry point for teachers to examine student thinking and consider their own beliefs about the teaching and learning of mathematics and also illuminated the need for professional learning opportunities beyond those available through the curriculum.

Frakes, C. & Kline, K. (2000). Teaching young mathematicians: The challenges and rewards. *Teaching Children Mathematics, 6*(6), 376. URL: <http://www.findarticles.com/p/articles/mi_hb3451/is_200002/ai_n8219691>.

This article provides an overview of a professional development project designed for a group of kindergarten teachers implementing the *Investigations in Number, Data, and Space* curriculum. Two main questions are posed to the teachers: 1)

How does a teacher recognize young mathematicians? 2) What is the teacher's role in developing mathematicians? The article explores one teacher's reflection on these questions and chronicles her changing vision of what it means to teach and learn mathematics.

Grant, T., Kline, K., & Van Zoest, L. (2001). Supporting teacher change: Professional development that promotes thoughtful and deliberate reflection on teaching. *NCSM Journal of Mathematics Education Leadership, 5*(1), 29–37.

The authors describe a professional development session they designed over the course of two long-term Local Systemic Change (LSC) grants. In implementing innovative mathematics curricula, teachers involved in the LSC programs struggled with eliciting, and engaging with students' mathematical ideas. The resulting "Reflecting on Teaching" session used videotapes of lessons taught by colleagues in order to engage teachers in thinking about the launch of a lesson, the support students need during their work, and the closure of the lesson.

Isaacs, A. (1997). Teacher development and *Everyday Mathematics. TeacherLink Newsletter, 6*(1), 1–3. URL: <http://www.wrightgroup.com/download/em/tl_fall_1997.pdf>.

This short article reviews the approach to professional development taken by a developer of the *Everyday Mathematics* elementary curriculum. Isaacs discusses various leadership development and in-service programs offered by the curriculum developers and publishers. Isaacs also introduces *Bridges to Classroom Mathematics*, a professional development project that assists in the implementation of the NSF-funded *Standards*-based elementary curricula. Also mentioned are grade-level videotapes that were available through the publisher of *Everyday Mathematics*, the Everyday Learning Corporation. (More information about the *Bridges* professional development program, which also contains materials to support the NSF-funded elementary curriculum *Investigations in Number, Data, and Space*, is available at http://www.comap.com/elementary/projects/bridges/index.htm)

Kazemi, E., & Franke, M. L. (2003). *Using student work to support professional development in elementary mathematics*. Seattle, WA: Center for the Study of Teaching and Policy. URL: <http://depts.washington.edu/ctpmail/PDFs/Math-EKMLF-04-2003.pdf>.

Building on the notion that embedding professional development within teachers' practice is a critical element of successful professional development, Kazemi and Franke describe an example in which the examination of student work provides the opportunity for teacher learning. The authors believed that looking at student work would provide an opportunity for teachers to raise questions about their practice, to carefully consider what they wanted students to learn, to understand their students' thinking, and to make sense of students' strategies. Through monthly workgroup meetings and classroom visits, Kazemi and Franke worked with eleven K–4 teachers to dissect student work and create opportunities to ponder both mathematical and pedagogical questions. Although the authors built on their knowledge of Cognitively Guided Instruction (CGI) and used the CGI framework as a basis for introducing terminology and strategy classification, they offer a framework for professional development that could be adapted to any curricula, as the student work from the teachers' classrooms and the needs of the teachers provided the structure for the workgroup discussions. For this group of teachers, engaging in the examination of student work revealed a trajectory of teacher learning. Initially teachers held rather traditional beliefs about the teaching and learning of mathematics. Rather than questioning teachers' practices, the facilitators created problems that would allow teachers to question their own practices and ask what classroom practices inhibited or promoted student learning. Through the workgroups the teachers also learned how to examine student work. At first, the teachers rarely engaged in discussions with their students about their strategies. The facilitators pushed the teachers to focus on the details of the student work, compare various strategies, consider next steps, and truly understand what the student was doing. As teachers examined student work in greater detail they created a community with one another, engaged more deeply in the mathematics, and questioned their own practices. Using vignettes from the workgroup meetings, Kazemi and Franke share this process of change and the accompanying struggles teachers faced as they worked to understand their students' mathematical thinking.

Kent, L., Pligge, M., & Spence, M. (2003). Enhancing teacher knowledge through curriculum reform. *Middle School Journal, 34*(4), 42–46.

Standards-based curricula for the middle grades include new content, content that many teachers, particularly those who are elementary certified, may not find familiar. How can teachers increase not only their content knowledge, but their *pedagogical* content knowledge—the knowledge required for teaching this new content so that students understand it? Kent, Pligge, and Spence propose that the obvious solution, additional graduate level courses, may not be the best one. Instead, they look to research showing that teacher content knowledge can actually be increased through the use of *Standards*-based materials. This model of professional development, "curriculum enactment," was used to help teachers learn new mathematics content in the context of their daily work. The authors found that staff development workshops focusing on new content and how it might be taught allowed teachers who were using an NSF-funded curriculum to learn necessary pedagogical content knowledge in concrete, non-threatening ways. Furthermore, once they were back in

the classroom, teachers were better able to understand the deeper mathematics present in the alternative problem-solving strategies their students used.

Krebs, A. & Burgis, K. (2003). Using *Standards*-based curriculum materials: A professional development model. *NCSM Journal of Mathematics Education Leadership, 6*(2), 8–12.

Krebs and Burgis outline a professional development model designed to support middle school teachers' implementation of *Standards*-based materials and increase their content and pedagogical content knowledge. After originally questioning what it takes to implement *Standards*-based materials, the authors investigated whether the original workshop participants were still implementing a reform curriculum five years after the initial year of professional development. This question was of particular interest because many of the original 34 participants taught in low-income schools, and research has suggested the importance of changed practice and ongoing support if implementation of *Standards*-based material is to be successful. Fourteen teachers were available for interviews that focused on their professional growth and on implementation issues. All reported that their districts were still using *Standards*-based materials. The interviews also revealed growth in the teachers' content and pedagogical content knowledge, leading them to feel they had become better teachers. Although current use of a *Standards*-based curriculum cannot be directly attributed to the original workshop, the participants did report that their initial year of professional development provided them with the necessary base for supporting implementation.

Kulm, G. (2003). *Improving mathematics teacher practice and student learning through professional development*. Paper presented at the Research Presession of the National Council of Teachers of Mathematics Annual Meeting, San Antonio, TX. URL: <http://www.coe.tamu.edu/~gkulm/NCTMpresessionpaper2003.doc>.

This article describes a study that is currently in progress. Designed to fill a gap in professional development research, Kulm illustrates the importance of addressing the interaction of multiple factors, including curriculum materials, instruction, professional development, and ongoing support when looking at issues that affect student learning and teacher practice. The author provides an overview of relevant research findings: how students learn, how teachers gain knowledge, and the kinds of professional development that can lead to teacher change and sustainability. Drawing heavily on the American Association for the Advancement of Science's (AAAS) Project 2061, the study looks at districts in Texas and Delaware that are implementing four of the middle school mathematics curricula that were analyzed and rated by the AAAS. Particular areas of interest include how teachers enact these curricula, and whether the highest rated materials (*Connected Mathematics* and *Mathematics in Context*) require much professional development support. During the first year of the project professional development focused on identifying learning goals and corresponding activities from the selected curricula and state standards. The second year was devoted to understanding how the curriculum materials support teachers within the context of the selected learning goals. Years 3–5 will focus on linking student achievement to teacher practice through multiple modes of assessment (including state standardized tests), and will use videotaped lessons and classroom observations to describe the enacted curriculum. Included in the article are the Project 2061 questions that will be used to analyze instructional practices and the use of curriculum materials in the videotaped lessons.

Lappan, G. (2000). A vision of learning to teach for the 21st century. *School Science and Mathematics, 100*(6), 319–326.

This article examines the challenges and changes confronting professional growth and teacher preparation programs for the 21st century. Throughout the article, Lappan explores the dilemmas associated with supporting teachers to continue learning their craft and to adjust to the changes implicit in mathematics reform. Reinforcing the view of the NCTM *Standards* that problem-solving and inquiry are central elements in reform curricula, the author suggests that teacher development programs should prepare teachers to be problem solvers and flexible thinkers. The article proposes three characteristics for successful professional development programs: 1) Student learning is the goal; 2) Professional development is strongly connected to the curriculum and standards for which teachers are accountable; and 3) Professional development builds on and asks teachers to examine their existing beliefs and knowledge.

Meyer, M. (2004). New tricks for old dogs. *Mathematics Teaching in the Middle School, 10*(1), 6–7. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2004-08-6a&from=B>.

In this short "On My Mind" article, Meyer discusses why learning to teach new, *Standards*-based mathematics curricula challenges all teachers, from new to experienced. She highlights some of the new expectations for teachers: using materials that have an unfamiliar format and sometimes different approach to the mathematics than more traditional materials; working on problems with colleagues, as if they are students; planning and assessing student learning collaboratively; and communicating with skeptical or angry parents who are having difficulty helping their children with homework. In short, she compares the experience to that of being a new teacher, but uses that comparison to suggest that the experience can be rejuvenating and exciting for teachers who enter the endeavor with open minds.

Parr, R., Papakonstantinou, A., Schweingruber, H., & Cruz, P. (2004). Professional development to support the NCTM *Standards*: Lessons from the Rice University School Mathematics Project's Summer Campus Program. *NCSM Journal of Mathematics Education Leadership, 7*(1), 3–12.

This article features the Rice University School Mathematics Project (RUSMP) Summer Campus Program for K–12 teachers. An example of a strong model of professional development that could be adapted for use with many curricula, this 4-week intensive program focused on improving teachers' mathematical content knowledge while promoting an understanding of the pedagogy and ideas in the NCTM *Standards*. The guiding principle behind the program is the fundamental belief that "sustaining wide-scale instructional reform can only be accomplished through the development of the skills and knowledge of individual teachers." Originally designed to bridge the gap between research and practice and as part of the partnership between Rice University mathematics faculty and Houston Public School teachers, RUSMP is committed to building teacher leaders who will be able to facilitate, design, and run the summer workshops.

Rodriguez, B. F. (2000). *An investigation into how a teacher uses a reform-oriented mathematics curriculum*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC document #ED 440868).

Abstract: This study examined the tensions that surfaced as a result of a teacher's transition from traditional to reform-oriented pedagogy while participating in the Mathematics Teacher Development Project and using the *Investigations in Number, Data, and Space* curriculum. In particular, the purpose of this study was to explore the relationship that develops between a teacher's learning in a teacher development project and her implementation of an innovative curriculum as it may influence the resolution of dilemmas that arise as she attempts to reform her practice. The study was conducted as a case study with a practicing teacher (Maria) who had already taught at the elementary level for 10 years. At the time of the study, Maria participated in the fifth (last) semester of a reform-oriented teacher development program and was using a reform-oriented curriculum for the first time. This study indicates that an effective teacher education program utilized in conjunction with a reform-oriented curriculum provides teachers with learning opportunities that promote the transformation of their traditional ways of teaching. However, their learning creates tensions and dilemmas resulting from specific conflicts (e.g., decision-making about the focus of lessons) between their evolving, reform-oriented understandings and the traditional ways to which teachers are accustomed.

Russell, S. J. (1996). The role of curriculum in teacher development. In S. Friel & G. Bright (Eds.), *Reflecting on Our Work: NSF Teacher Enhancement in K–6 Mathematics*. Lanham, MD: University Press of America, Inc. URL: <http://investigations.terc.edu/relevant/RoleOfCurriculum.html>.

This article explores several different attitudes toward curriculum and teachers' relationships with curriculum: as "teacher-proof," as an aid for less-prepared teachers, or as reference material. Russell reasons that curriculum is best used as "a tool that allows the teacher to do her best work with students" and that the best teaching environment is a partnership between a teacher and the curriculum in use. She describes ways teachers can use curriculum as a development tool to further their own thinking about working with students, and cites examples of this teacher-curriculum partnership in action. Russell emphasizes that a curriculum does not hold all the answers or foolproof methods, and that the best-designed curriculum must assume that what it suggests won't always work. The article concludes with examples of ways the curriculum can be used as a development tool for both in-service and pre-service teachers.

Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education, 1*(1), 55–87.

This article focuses on three teachers who were trying to teach fractions, and *Teaching to the Big Ideas* (TBI), a professional development project in which two of them were involved. Although all three had tried to make student thinking and communication central in their classrooms they still struggled when thinking about their students' understanding of fractions and the questions that they were asked. In addition they recognized a need to address their own comfort level when thinking and talking about the topic. The article discusses how TBI helped focus teachers on making inquiry into mathematics, first as learners and then as teachers. The theory was that teachers would be better able to respond to students' questions and conceptual understandings if they themselves had thought about various mathematical concepts in more complex ways. By exploring transactions in two of the teachers' classrooms in depth, the article helps readers understand how deeply teachers need to think about mathematical ideas in order to effectively facilitate classroom discussions about them.

Teacher Practice

Bolster, L. C., & Reys, R. E. (2002). Modeling middle school mathematics: A technology-based professional development resource. *Journal of Mathematics Education Leadership, 6*(1), 36–39. URL: <http://www.mmmproject.org>.

How might a district help teachers become interested in and knowledgeable about *Standards*-based materials and practices? Ideally, teachers would talk with colleagues who had implemented *Standards*-based curricula in nearby districts and observe lessons using these materials. Even if this first-hand opportunity is lacking, however, teachers can still get an in-depth look at *Standards*-based lessons through *Modeling Middle School Mathematics* (MMM), a video project sponsored by the Show-Me Center, the National Center for *Standards*-based Middle Grades Mathematics Curricula (http://showmecenter.missouri.edu). The MMM website (http://www.mmmproject.org) hosts ten full-length lessons that showcase five *Standards*-based middle school curricula and address all five NCTM content strands. This comprehensive online package features video streaming of complete lessons, full transcripts of each lesson, examples of student work, interviews with teachers about implementation concerns, and interviews with program developers who explain each curriculum's vision of teaching and learning. In addition to being taught in diverse settings, the lessons allow for the personality of the teacher to shine through, helping viewers see how a *Standards*-based curriculum can be taught with both a high degree of fidelity and a personal touch. In addition to being useful during the textbook selection and adoption process, MMM materials (VHS tapes of the lessons are also available) could be used as a professional development resource during the implementation phase, or utilized with administrators, parents, and community stakeholders to help explain what a *Standards*-based curriculum looks like in practice.

Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *The Elementary School Journal, 103*(3), 287–311.

Outlining the key factors of professional development (e.g., ongoing, opportunity to examine and build beliefs and knowledge about teaching and learning, linked to the context of the classroom, etc.), Collopy posits that curriculum materials could potentially incorporate these elements of effective professional development. While professional development experiences often use curriculum materials as part of teacher learning, the author questioned whether (and what) teachers might learn solely from *Standards*-based curriculum materials that incorporate support for teacher learning. In order to investigate this question, Collopy studied two experienced upper-elementary teachers piloting *Investigations* after having used a more traditional text. Both teachers attended a two-day workshop introducing them to the materials but had no further professional development other than their new curriculum. Through observations and interviews, Collopy collected baseline, ongoing, and end of the year data about the teachers' use of the curriculum materials and their beliefs and knowledge about the teaching and learning of mathematics. Although both teachers participated in the same introductory workshop and had access to the same materials, their opportunities for learning differed because of the way each teacher enacted the curriculum and interpreted its content with a different lens. The first teacher, who felt confident with her many years of teaching experience and mathematical knowledge, maintained her beliefs that children needed to know the basics. Her instructional practices revealed only surface changes, and her selective use of the materials and continued emphasis on procedures and correctness eventually caused her to abandon the use of *Investigations* midway through the school year. In contrast, the other teacher wholeheartedly embraced the *Investigations* materials, followed the guidelines given for teachers, learned valuable teaching strategies, and developed trust in the curriculum. Over an extended period of time her beliefs about mathematics instruction shifted from a focus on procedures and correctness to mathematical understanding and processes. As a way to illuminate these results, Collopy provides lesson excerpts to illustrate how each teacher incorporated *Investigations* into her instructional practice. The author also notes that given the varying use of curriculum materials it seems necessary to recognize the strong connection between teacher beliefs and identity, and to consider beliefs both as influences and as targets for change.

Dugdale, S., Matthews, J. I., & Guerrero, S. (2004). The art of posing problems and guiding investigations. *Mathematics Teaching in the Middle School, 10*(3), 140–147. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2004-10-140a&from=B>.

Implementing reform curricula as intended by the developers often requires teachers to dramatically change their pedagogy and for both teachers and students to assume roles different from those traditionally associated with mathematics. In this article, the authors offer an example of what these key aspects of pedagogy might look like in a classroom by examining one teacher's (Mr. Hill) use of questioning as he focuses on problem-solving. Dugdale, Matthews, and Guerrero provide excerpts of dialogue that show a progression of learning and specific questions that foster understanding. Key elements of Mr. Hill's pedagogy include establishing students' ownership of a problem, creating a safe environment, constant assessment of students' understanding, and advancing students' exploration

to the next stage of understanding. Knowing what questions to ask and how to actively involve and challenge all students is a complex aspect of reform. However, this article provides guidelines for the types and roles of different kinds of questions and describes how one teacher successfully develops the habits of mind (e.g., listening to one another, questioning) of his students to foster mathematical problem solving.

Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in *Everyday Mathematics* classrooms. *Journal for Research in Mathematics Education, 30*(2), 148–170. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME1999-03-148a&from=B>.

From a study of 18 experienced, first-grade *Everyday Mathematics* teachers, researchers developed a framework for Advancing Children's Thinking (ACT), which is designed to be used by educational researchers, teacher educators, and curriculum designers. The ACT framework was based upon the exemplary pedagogy of a teacher whose practice fell into three distinctive patterns: eliciting children's solution methods, supporting children's conceptual understanding, and extending children's mathematical thinking. The article uses observation and interview data to illustrate ways this teacher successfully achieved these three components in her *Standards*-based classroom. The authors also provide an analysis of the practice of all 18 teachers participating in the study.

Frykholm, J. & Pittman, M. (2001). Fostering student discourse: "Don't ask me! I'm just the teacher!" *Mathematics Teaching in the Middle School, 7*(4), 218–221. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2001-12-218a&from=B>.

Citing changes in curricula in response to the NCTM *Standards*, the authors argue for changes in teachers' facilitation of student-directed investigations in mathematics classrooms. This article outlines pedagogical changes that teacher Mary Pittman made as part of her implementation of *Mathematics in Context*, such as having students take notes during classroom discussions and add their peers' solution strategies to their own completed homework assignments. She also created a list of questions that encouraged greater student dialogue. Most importantly, she tuned in to "her own perceived need to be the mathematical authority in the classroom." The article ends by offering tips to help teachers make changes in their practice in order to provide an energetic yet safe environment in which students can construct understanding and share their thinking with one another.

Isaacs, A. & Carroll, W. (1999). Strategies for basic facts instruction. *Teaching Children Mathematics, 5*(9), 508. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=TCM1999-05-508a&from=B>.

Isaacs and Carroll discuss the role of basic facts instruction in reform mathematics and propose strategies to help teachers recognize students' natural talents and develop their conceptual understanding. The authors encourage teachers to use the following strategies for helping students learn basic number facts: counting to solve problems; understanding parts and wholes; deriving facts from facts they know; and practicing skills in appropriate ways. The article also addresses assessment of students' fact knowledge, recommending a variety of assessment strategies that provide teachers with a thorough picture of students' knowledge.

Keiser, J. & Lambdin, D. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *The Journal of Educational Research, 90*(1), 23–31.

Abstract: Time issues raised by sixth- and seventh-grade teachers involved in field-testing an NSF-sponsored mathematics curriculum, *Connected Mathematics* (CMP), were examined in this study. Questions investigated included the following: How much scheduled time is actually available for mathematics instruction in elementary and middle schools and how is it configured? How do project teachers and students spend their time in class? What factors influence CMP teachers' pacing through this new curriculum? Findings indicate that teaching in the spirit of the current mathematics education reform movement may be highly dependent upon flexibility in class scheduling. Innovations in teaching mathematics (e.g., increased group work, writing, extended projects, and alternative forms of assessment) seem to require additional time and new ways of thinking about using class time.

Lambdin, D. & Preston, R. (1995). Caricatures in innovation: Teacher adaptation to an investigation-oriented middle school mathematics curriculum. *Journal of Teacher Education, 46*(2), 130–140.

This article examines teacher change due to reform efforts in mathematics education, taking a specific look at the *Connected Mathematics* program (CMP), an NSF-funded, *Standards*-based, middle school curriculum. The authors of this article were external evaluators during trials of the sixth-grade curriculum materials, and write about what they observed as they watched a variety of districts implementing CMP. The authors discuss the methodology of the program and what they observed—a commonality among teachers characterized by three main classifications of teachers: the "Frustrated Methodologist," the "Teacher on the Grow," and the "Standards Bearer." Each catagory of teacher is based on interviews with teachers and observation data, and is illustrated by a caricature.

Lappan, G. (1997). The challenges of implementation: Supporting teachers. *American Journal of Education, 106*(1), 207–239.

Abstract: Reform in mathematics education has been

stimulated and propelled by the publication of standards documents by the National Council of Teachers of Mathematics (NCTM). This article examines the vision of teacher decision-making that is portrayed in the NCTM *Professional Standards for Teaching Mathematics:* choosing worthwhile mathematical tasks, orchestrating and monitoring classroom discourse, creating an environment for learning, and analyzing one's practice. The philosophical orientation and the set of commitments to teaching and learning on which the *Standards* are based include stances on equity, curriculum, teaching, and learning. These stances are summarized under the following headings: inclusiveness, depth of coverage, teaching for understanding, active engagement of students, and curriculum investigations, applications, and connections.

Manouchehri, A. & Goodman, T. (1998). Mathematics curriculum reform and teachers: Understanding the connections. *The Journal of Educational Research, 92*(1), 27–41. URL: <http://static.highbeam.com/t/thejournalofeducationalresearch/september011998/mathematicscurriculumreformandteachersunderstandin/>.

Abstract: Ethnographic research was conducted to study the process of evaluation and implementation of four *Standards*-based curricular materials by 66 middle school mathematics teachers at 12 different school districts over a period of two years. The data revealed that what teachers knew about mathematics content and innovative pedagogical practices and their personal theories about learning and teaching mathematics were the greatest influences on how they valued and implemented the programs. Moreover, the environments within which teachers worked were instrumental in their use of the materials. The problems teachers faced as they taught the curriculum included lack of sufficient time for planning, lack of conceptual understanding of mathematics concepts, inadequate knowledge base about how to bridge the gap between teaching for understanding and mastery of basic skills, and lack of professional support and progressive leadership.

Manouchehri, A. & Goodman, T. (2000). The challenge within: Implementing mathematics reform. *Educational Studies in Mathematics, 42*(1), 1–34.

This research is a case study of two seventh-grade mathematics teachers as they implement a *Standards*-based curriculum. The study investigates ways in which the teachers facilitated the use of the new curriculum in their classrooms and examines impediments to implementation. The study, which draws on two years of interviews and observational data, focuses on the teachers' interactions with the curriculum along with their reflections on their practice.

Meyer, M. (1999). Multiple strategies = multiple challenges. *Mathematics Teaching in the Middle School, 4*(8), 519–523. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-05-519a&from=B>.

What implications does exploring a problem in a *Standards*-based mathematics curriculum have for teachers and students? Meyer examines several strategies students might use to solve a problem from a sixth-grade algebra unit in *Mathematics in Context* (MiC): using expressions with variables, guessing and checking, or creating a combination chart. The author then discusses five teacher challenges associated with encouraging multiple strategies in the classroom: 1) welcoming multiple strategies; 2) recognizing students' different mathematical understandings within their strategies; 3) deciding what instruction is suggested by these different strategies; 4) managing changes in the social order of the classroom; and 5) responding when one doesn't understand a student's solution. The article closes by encouraging teachers to accept these challenges as pathways toward a more exciting mathematics classroom.

Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientation toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*(5), 352–388. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2004-11-352a&from=B>.

With the goal of understanding the role of curriculum materials in supporting teacher learning, Remillard and Bryans studied eight elementary teachers implementing *Investigations* in a predominately low-income, African American urban school. Specifically, the authors considered how these teachers engaged with and enacted the curriculum, and how their use of the materials provided varying learning opportunities. This two-year study involved classroom observations, interviews, and monthly meetings in a school where there was emphasis on and support for professional development. Analyses occurred individually and then collectively, and focused on teachers' beliefs about and perceptions of the curriculum, teaching, and learning, as well as classroom practices. Remillard and Bryan described teachers' use of the materials thorough curriculum mapping, curriculum design, and enacted curriculum (See Remillard 1999 for a further description of this framework) and categorized teachers' use as intermittent and narrow, adopting and adapting, or through piloting. Substantial differences emerged among teachers' orientation toward the curriculum. Interestingly, teachers could share similar views about teaching and learning but use curriculum materials quite differently. Likewise, teachers with contrasting beliefs about teaching and learning could use the curriculum very similarly. The authors then describe how teachers' use of curriculum materials relates to different learning opportunities (e.g., insights into student thinking, exploration of mathematics, etc.). Although teachers' perspectives and use of the curriculum remained stable, relatively new teachers exhibited more changes and were more likely to be faithful to the reform curriculum. This article highlights the complexity of the relationships between teachers and curriculum and the need to further examine teachers' orientation and use of curriculum materials.

Reys, B. & Reys, R. (1997). *Standards*-based mathematics curriculum reform: Impediments and supportive structures. *NCSM Journal of Mathematics Education Leadership, 1*(2), 3–8. URL: <http://www.ncsmonline.org/NCSMPublications/1997journals.html#jul97mel>.

This article describes the outcomes of a three-year teacher enhancement effort that provided a forum for the study and implementation of *Standards*-based mathematics curricula. The project worked with teachers as they reviewed and tested curriculum materials, and used this process as a "vehicle for teacher enhancement" that "prompted many issues, including assessment, teaching practices, and content-related questions, to emerge." Reys and Reys cite several lessons that the project learned through this process: that collaborative curriculum investigation is a powerful force not only in helping teachers in teams move toward the *Standards*' vision, but also in professional development; how a well-constructed and diverse team of reviewers (teachers, administrators, parents) can provide leadership; how a multi-year commitment to this investigation and subsequent implementation provides knowledge development and professional growth; the importance of addressing potentially controversial issues (including ways to inform parents, assessment of student learning, and alignment of elementary, middle, and secondary curricula); and the value of establishing an electronic communication network for teachers experimenting with *Standards*-based curricula.

Ross, J., McDougall, D., & Hogaboam-Gray, A. (2003). A survey measuring elementary teachers' implementation of *Standards*-based mathematics teaching. *Journal for Research in Mathematics Education, 34*(4), 344–363. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2003-07-344a&from=B>.

To examine the extent to which teachers actually implement reform-oriented mathematics, the authors developed a 20-item survey based on nine dimensions of *Standards*-based teaching, and then correlated survey results to both classroom observations and scores on a mandated student performance assessment. The beginning of the article emphasizes survey design, research protocol, and reliability/validity of the instrument. The latter portion presents the authors' findings: that teachers who are similar in their claims about using a *Standards*-based text may differ in how they use the text, and that teachers using reform-oriented curricula are not necessarily reform-oriented teachers. The authors found that teachers using *Standards*-based materials could be categorized as High-reform or Low-reform. High-reform teachers were found to use *Standards*-based texts as "amplifiers" of their practice, whereas Low-reform teachers used the text as a "justifier," adopting the language of reform but not its substance. Although they used the reform curriculum's activities, low-reform teachers transformed them in subtle ways so that traditional practices were maintained. (The authors' nine-dimension "Rubric for Implementation of Elementary Mathematics Teaching" is included in the article and might be useful for supervisors or teacher educators when describing characteristics of reform-oriented practice they hope to see in classrooms.)

Silver, E. & Smith, M. S. (1997). Implementing reform in the mathematics classroom: Creating mathematical discourse communities. *Reform in Math and Science Education: Issues for Teachers*. Columbus, OH: Eisenhower National Clearinghouse. URL: <http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-004816-4816,00.shtm>.

In this article, Silver and Smith refer to three of the six NCTM *Standards* that deal with "notions of classrooms as discourse communities and of teachers as facilitators of mathematical discourse," and discuss how creating such an environment would look in its implementation. To illustrate the implementation of this notion, Silver and Smith describe a mathematical discourse scenario dealing with ratio and area in a seventh-grade urban classroom. Taking a closer look at this case, the article explores three important factors: getting students to talk by encouraging discourse, giving students something to talk about by giving them worthwhile problems, and helping students learn to talk about mathematics by effectively monitoring and supporting the discourse. Some ways to support teachers in this effort are suggested, including discourse among colleagues, outside resources, and personal reflection. The support and implementation methods cited use examples from the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project.

Teacher Perspectives

Lloyd, G. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education, 2*(3), 227–252.

Abstract: This paper describes two high school teachers' conceptions of the cooperation and exploration components of a reform-oriented mathematics curriculum [the Core-Plus Mathematics Project]. Although the teachers appreciated the themes of cooperation and exploration in theory, their conceptions of these themes with respect to their implementation of the curriculum differed. One teacher viewed the curriculum's problems as open-ended and challenging for students, whereas the other teacher claimed that the problems were overly structured. Each teacher attributed difficulties with students' cooperative work to the amount of structure and direction (too little or too much) offered by the problems. Discussion of such similarities and differences in the teachers' conceptions emphasizes the dynamic, humanistic nature of curriculum implementation and gives rise to important implications for mathematics teacher development in the context of reform.

Meyer, M. & Ludwig, M. (1999). Teaching with MiC: An opportunity for change. *Mathematics Teaching in the Middle School, 4*(4), 264–269. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-01-264a&from=B>.

Matthew Ludwig reflects upon changes he has had to make as a teacher shifting from a traditional mathematics curriculum to the *Standards*-based *Mathematics in Context* (MiC) curriculum. Margaret Meyer reacts from the perspective of an MiC developer who has worked with several teachers as they assume new roles through the use of this curriculum. Meyer and Ludwig offer their perspectives on the roles of Teacher as Leader, Teacher as Colleague and Colearner, Teacher as Communicator with Parents, and Teacher as Teacher.

Pligge, M., Kent, L., & Spence, M. (2000). *Examining teacher change within the context of mathematics curriculum reform: Views from middle school teachers.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC document #ED 443726).

Abstract: This article describes teacher change using the backdrop of a *Standards*-based reform mathematics curriculum for middle grades and direct quotes from teachers and math support cooridinators involved in the implementation over the last five years. Each of the 16 participants had at least one year of experience teaching or supervising the instruction of the curriculum, *Mathematics in Context*, which was designed to build instruction on students' informal knowledge using meaningful context situations. The results of survey, interview, and classroom observation data illustrate that, for these selected teachers and support staff, both the design of the curriculum and the staff development workshops changed their perceptions of both what mathematics is as a subject and how mathematics should be taught. The quotes from the participants and the specific examples that they refer to in the curriculum provide evidence of the legitimacy of their perceptions about how they have changed their beliefs about teaching and learning as a result of their interactions with this particular curriculum. The paper concludes with a discussion of the impact of the reform in general and the continued vision shared by mathematics educators to help all students learn significant mathematics.

Stevens, B. A. (2001). My involvement in change. *Mathematics Teaching in the Middle School, 7*(3), 178–182. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2001-11-178a&from=B>.

This article, the reflections of a fourth-year teacher, examines the author's experience implementing the middle school curriculum *Mathematics in Context* in a school that had provided substantial support to its teachers. Stevens comments on the professional relationships among teachers as well as the administrative support for implementation. A significant portion of the article outlines changes in the author's instructional practice as she used the program in a well-structured school environment.

Tetley, L. (1998). Implementing change: Rewards and challenges. *Mathematics Teaching in the Middle School, 4*(3), 160–165. URL: <http://showmecenter.missouri.edu/showme/publications/ic/ic.html>.

Tetley reflects on her experience as a mathematics teacher undergoing changes associated with the implementation of a new curriculum, *MATH Thematics*. She describes the benefits of the training she experienced prior to implementation, as well as some of the differences between *MATH Thematics* and materials she had used in the past. Tetley voices her initial concerns, including the challenge of adjusting to new forms of assessment, but now views the changes brought on by the new curriculum as positive. She recommends that others also experience implementing a *Standards*-based curriculum.

Van Boening, L. (1999). Growth through change. *Mathematics Teaching in the Middle School, 5*(1), 27–33. URL: <http://www.showmecenter.missouri.edu/showme/publications/mtmsGROWTH.html>.

Van Boening, a middle-grades teacher in a small and isolated

rural community, relates her experience adopting the *MATH Thematics* curriculum for her classroom. She reviews her process of change, support structures that were needed, major shifts that occurred, and how she looked ahead to her fifth year of teaching the program. The change process was a gradual one, and Van Boening shares examples of lessons used as well as her own reflections.

Pre-service Learning

Chval, K. (2004). Making the complexities of teaching visible for prospective teachers. *Teaching Children Mathematics, 11*(2), 91–96. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=TCM2004-09-91a&from=B>.

In this article, Chval describes an assignment for an elementary mathematics methods course that was based on the design and findings of a National Science Foundation (NSF) Study involving prospective teachers. The NSF study emerged from the belief that prospective teachers need the opportunity to practice and analyze their own teaching. The study analyzed the effects of prospective elementary school teachers repeatedly teaching the same lesson to a small group of students as well as conducting the same interview with multiple children. Both the repeated lessons and interviews afforded these pre-service teachers the chance to focus on their students' development of mathematical concepts and to reflect on how their instructional choices affected student understanding. NSF-supported curricula, such as *Investigations*, *Connected Mathematics*, and *Math Trailblazers* were used for all the lessons. Prospective teachers reported that these experiences allowed them to better anticipate student responses and struggles, modify instruction, understand that the lesson might go in a different direction depending on the students in the group, and value the importance of language and discourse. Recognizing the value of prospective teachers' discussing and analyzing their own teaching, Chval created an assignment in which her students interviewed three children using the same questions. Following preparatory work in class, prospective teachers were responsible for designing, conducting, and analyzing their interviews, all processes that they found very challenging. Despite these reported challenges, Chval's interviews with her students revealed that conducting the interviews allowed prospective teachers to see the complexity in developing, understanding, and assessing children's mathematical knowledge, as well as the value of letting students solve problems on their own.

Hodgson, T. & Riley, K. (2001). Real-world problems as contexts for proof. *Mathematics Teacher, 94*(9), 724–729. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MT2001-12-724a&from=B>.

This article explores pre-service middle and high school teachers' work on a unit focused on the mathematics of reflection, using materials from the *Standards*-based secondary curriculum, *SIMMS Integrated Mathematics* and from *Geometry: An Integrated Approach*. The authors suggest that when working on problems based in real-world contexts, students develop mathematical reasoning and ultimately, proof. These pre-service students first solved their reflection problem using an algorithm. However, when testing their solution by modeling the problem in a real-world context, they discovered an error. The students worked carefully within the problem context to refine their model and ultimately reached a correct solution. More importantly, in so doing, they developed a proof to support their rationale.

Lloyd, G. & Frykholm, J. (2000). On the development of "book smarts" in mathematics: Prospective elementary teachers' experiences with innovative curriculum materials. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, 2. Retrieved December 8, 2004, from http://www.k-12prep.math.ttu.edu/journal/pedagogy/lloyd01/article.pdf.

Teacher educators face a variety of dilemmas. How can they build a vision of *Standards*-based mathematics for pre-service teachers who have a weak and narrow knowledge of mathematics based on years of experience as students in traditional classrooms? If teachers, not texts alone, determine how innovations become implemented in the classroom, how can pre-service teachers be exposed to new instructional practices? And, "Given the extremely brief time period in which teacher educators interact directly with pre-service teachers...." how can professors introduce new content knowledge and then turn that knowledge into *pedagogical* content knowledge? Lloyd and Frykholm addressed these issues by having their elementary pre-service teachers use middle school *Standards*-based curricula such as *Connected Mathematics* (CMP) and *Mathematics in Context* (MiC). Through the use of these curriculum materials, the prospective teachers learned unfamiliar geometry concepts in a manner that allowed them to "make connections between their own difficulties and those of their envisioned future students." In addition, the authors had their students read an NCTM document on geometry—but only *after* having done the hands-on geometry activities. They note, "Because the reading of the NCTM document came immediately after working on the CMP unit, the bulk of the student comments integrated ideas from both books...reading the *Standards* document helped them make better sense of what they had done during the (CMP) unit." Student comments are woven throughout the article, giving the reader a sense of the how this model of learning influenced these pre-service teachers. It also suggests potential for using a similar framework as a method of professional development for veteran teachers who need to increase their pedagogical content knowledge in a timely fashion.

Papick, I., Beem, J., Reys, B., & Reys, R. (1999). Impact of the Missouri Middle School Mathematics Project on the preparation of prospective middle school teachers. *Journal of Mathematics Teacher Education, 2*(2), 301–310. URL: <http://www.ingentaconnect.com/content/klu/jmte/1999/00000002/00000003/00238924;jsessionid=51gllrscaejfd.victoria>.

Based on the findings of an extended middle-grades professional development project grounded in *Standards*-based mathematics curricula, this article explains how the authors worked with colleagues to revamp the Middle Grades Mathematics Certification program at the University of Missouri. The Missouri Middle Mathematics (M^3) Project (1995–1999) worked to provide curriculum-focused professional development to in-service teachers, and in doing so, recognized new needs for pre-service teachers. They addressed these needs in their pre-service program by instituting changes in the mathematical content of the teacher preparation program as well as creating a newly-designed partnership between the Department of Mathematics and the Department of Curriculum and Instruction.

Spielman, L. J., & Lloyd, G. M. (2004). The impact of enacted mathematics curriculum models on prospective elementary teachers' course perceptions and beliefs. *School Science and Mathematics, 104*(1), 32–44. URL: <http://www.math.vt.edu/people/lloyd/publications/scans/Spielman_Lloyd_04_SSM_scan.pdf>.

Given the vision of curriculum and instruction described in NCTM's *Principles and Standards for School Mathematics*, the authors address the importance of prospective teachers' knowledge of and beliefs about the teaching and learning of mathematics. Spielman and Lloyd cite the challenges in translating ideas of reform into practice, and the tendency of prospective teachers' beliefs and practices to reflect their own schooling experiences. In their study, they examined how the use of two different curricular models used in a mathematics methods course affected prospective elementary teachers' content knowledge and beliefs. One section of the course drew heavily on units from *Mathematics in Context* and *Connected Mathematics* where the focus was on prospective teachers initiating and supporting class discussions. The other section relied on a more traditional methods text, and both the text and the instructor served as mathematical authorities. Neither of the sections explicitly addressed pedagogy with students, but both the teaching methods and course design reflected the philosophy of the authors' of the texts being used. Through the use of both pre- and post-survey instruments, Spielman and Lloyd found no difference in mathematical content knowledge between the two sections. Yet post-survey data suggest that the beliefs of students in the curriculum materials section changed more than those of their peers in the textbook section. The former group of prospective teachers valued exploration over practice, placing greater emphasis on classroom group work and discussion and less on the instructor's lectures and on textbooks filled with examples, explanations, and practice problems. Their resulting beliefs about instruction were much more closely aligned with principles of reform than often attributed to pre-service teachers, and highlight the explicit messages that can be acquired through implicit course design. While the results have limited generalizability, they do suggest that reform curricula, in this case *Mathematics in Context* and *Connected Mathematics* and instructional practices that truly support their intentions are attended to by prospective teachers and influence how they think about instruction.

Stump, S., Bishop, J., & Britton, B. (2003). Building a vision of algebra for pre-service teachers. *Teaching Children Mathematics, 10*(3), 180–186. URL: <http://www.findarticles.com/p/articles/mi_hb3451/is_200311/ai_n8218779>.

How can teacher educators prepare pre-service mathematics teachers to teach algebraic thinking in elementary and middle school when they themselves may have only a shallow understanding of the subject? The authors believe that their role is to challenge their students' existing views of algebra and to shift the focus from symbolic manipulation to the development of algebraic concepts. By organizing their pre-service curriculum around three categories—generalization, problem-solving, and functions and modeling—and by drawing from exemplary curriculum materials (i.e., *Mathematics in Context* and *Connected Mathematics*)—the instructors were able to bring out the power of algebra, showing future teachers that algebra is more than a body of rules and procedures. They encouraged their pre-service teachers to analyze and question, to reason and communicate their thinking, to explore relationships between numbers and variables, and to look at patterns of change. By viewing algebra from various perspectives and reflecting on questions that go beyond a problem's solution, pre-service teachers broadened their vision of algebra so as to become better equipped to teach mathematics effectively in their elementary and middle school classrooms.

Taylor, P. M. (2000). When are we ever going to use this? Lessons from a mathematics methods course. *School Science and Mathematics, 100*(5), 252–255.

Taylor, a teacher educator, addresses students' concerns about whether the objectives for his mathematics methods course are applicable to the "real" classroom. His course focused on teaching pre-service teachers to foster mathematical discourse and problem-solving, and to use cooperative learning strategies in their middle-grades mathematics classrooms. In order to address his students' concerns of not being able to use what they were learning in more traditional classrooms, Taylor decided to revise his course and help his students understand that even if handed a traditional text, they could still teach in a manner that was consistent with the NCTM *Standards*. Taylor and his students took the basic structure of a traditional text

and discussed the mathematical context of a unit. They then incorporated elements of *Standards*-based instruction and discussed types of assessment. Taylor helped his pre-service teachers simulate planning for and teaching in an environment that might not be enthusiastic about *Standards*-based instructional strategies and materials in order to bridge the disconnect prospective teachers often feel between methods classes and their classroom placements.

Van Zoest, L. & Bohl, J. (2002). The role of reform curricular materials in an internship: The case of Alice and Gregory. *Journal of Mathematics Teacher Education, 5*(3), 265–288.

Prospective teachers often experience a disconnect in philosophy between teacher education programs and their student teaching experience. In this article, the authors describe the experience of a secondary school mathematics teacher and her intern whose beliefs aligned with reform ideas called for by NCTM. The case outlined in this article tracks the working relationship between a reform-minded teacher intern and her like-minded mentor. The mentor teacher uses the Core-Plus program, and they rely on the curriculum to help them plan lessons and challenge their own thinking about how to help students recognize connections between the mathematics they are studying and the real world. Their conversations regularly focused on mathematics content, conceptual understanding and the use of questioning to further students' learning. The case follows the intern from her student teaching experience through her first year of teaching, where she reshapes the math program to embrace reform-minded principles and materials. The intern had the opportunity to use the Core-Plus materials in a supportive environment, which resulted in a positive experience that fostered her development and learning and gave her the confidence to create changes during her first year of teaching.

Wilson, S. & Ball, D. (1996). Helping teachers meet the *Standards:* New challenges for teacher educators. *The Elementary School Journal, 97*(2), 121–137.

Abstract: In this article, Wilson and Ball describe the curricular standards that currently concern teachers and teacher educators. They propose that these reforms pose three significant challenges for teacher educators. First, the reforms are based on changing images of good teaching. Second, the teaching that lies at the heart of the reforms is undetermined and uncertain. Third, there is little theory concerning how beginning teachers learn to teach in these ways. Given these three challenges, Wilson and Ball sketch several avenues for investigation that teacher educators may consider as they reformulate teacher education programs in this *Standards*-based era of educational reform.

Ziebarth, S., Slezak, J., Lagrange, D., & Kleinfelter, N. (1997). *Teaching a reformed high school mathematics curriculum: In-service and pre-service perspectives*. Paper presented at the Annual Meeting of the Association of Mathematics Teacher Educators, Washington, DC.

Each author of this paper presents his or her own individual perspective about the ways in which *Contemporary Mathematics in Context* (Core-Plus), a reform-oriented high school curriculum, requires teachers to learn new ways of thinking about mathematics and the teaching of mathematics. Each perspective highlights new challenges teachers face in teaching a program like Core-Plus, but the primary focus of the article is the implications these challenges have for pre-service teacher education, as well as professional development for in-service teachers. Topics touched upon include ways of working with cooperative groups, new understandings of mathematics topics and the connections between them, as well as preparation for answering students' questions about the applications of mathematics.

Leadership Support

Doyle, M. (2000). *Making meaning of teacher leadership in the implementation of a Standards-based mathematics curriculum*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans. (ERIC document #ED 448140).

This qualitative case study examines one district's use of teacher leaders in supporting the implementation of an elementary grades, *Standards*-based mathematics curriculum. The paper highlights findings from a study of four teacher leaders in two schools that were implementing *Investigations in Number, Data, and Space*. The study frames eight fundamental principles for providing effective professional development, including: enabling teachers to enhance their content and pedagogical knowledge, implementing curriculum materials that embody effective teaching and learning strategies, and continually assessing professional development to insure that it is meeting teachers' needs. The author also proposes seven leadership roles that make the work of teacher leaders effective and valuable to classroom teachers: providing moral support, gathering resources and materials, working as a liaison between teachers and administrators, creating a collaborative work atmosphere, mentoring teachers, providing instructional and content knowledge at staff meetings, and enabling others to lead. Doyle concludes that teacher leaders' work must be well supported in order to allow them to provide both professional development and managerial support to teachers.

Goldsmith, L. (2001). Spheres of influence: Supporting mathematics education reform. *National Association of Secondary School Principals (NASSP) Bulletin, 85*(623), 53–65.

The author argues that principals play a key role in supporting the changes that teachers and students undergo during the implementation of *Standards*-based mathematics in their schools. As instructional leaders, principals should understand the goals of mathematics education reform beyond the superficial level. Goldsmith provides concrete suggestions as to how principals can lead curricular change efforts in their schools by supporting both a challenging curriculum for students and professional development for faculty. The author asserts that principals not only need to become familiar with the resources available for improving mathematics instruction, but also to make firm commitments of resources to support the effort. As leaders of the school community, principals should play a primary role in helping parents and community members understand changes in mathematics instruction. Ultimately, Goldsmith challenges principals to create, and then stick to, a strategy for implementing reform.

Huinker, D. & Freckmann, J. L. (2004). Focusing conversations to promote teacher thinking. *Teaching Children Mathematics, 10*(7), 352–357. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=TCM2004-03-352a&from=B>.

This article compares two types of questioning techniques that supervisors, teacher educators, and staff developers might use with teachers during debriefing conferences after classroom observations. Rather than asking primarily about the mechanics and management of the lesson, which may lead to superficial responses, teacher educators and staff developers are encouraged to ask questions that focus on the goals of the mathematical task, on student understanding, and on teacher decison-making. A three-step process, based on the work of Garmston and Costa, for structuring questions to promote teacher thinking, deepen discourse, and reflect on practice is described.

Lester, J. B. & Miles-Grant, C. (2001). Mathematics supervision through a new lens. *Educational Leadership, 58*(5), 60–63.

This article describes *Lenses on Learning: A New Focus on Mathematics and School Leadership*, a course designed to help administrators become more effective mathematics supervisors. The article chronicles the experiences of an elementary school principal who entered the program feeling poorly-equipped to evaluate or support mathematics teaching. In describing the *Lenses on Learning* program, the article illustrates how administrators learn to support mathematics instruction based on the NCTM *Standards* by engaging in mathematics activities and exploring students' thinking and teachers' roles in *Standards*-based classrooms.

Malloy, C. (2003). The new math. *Principal Leadership, 3*(7), 48–53.

What do principals and supervisors need to know about the teaching and learning of mathematics in order to be effective instructional leaders in an era of curriculum change? More importantly, what new observation and supervision skills do they need? After highlighting some of the content changes in middle-school mathematics, Malloy describes a lesson on data analysis to help administrators answer the question, "What does a *Standards*-based classroom look like?" Rather than looking only for students with quick recall or algorithmic facility, the author notes five types of intellectual behaviors an administrator should expect to see in a *Standards*-based classroom: students who are 1) constructing relationships, 2) extending knowledge, 3) reflecting about experiences, 4) articulating what they know, and 5) making mathematical knowledge their own. Malloy continues by describing what an administrator should see in the teaching of mathematics: rigorous content, meaningful tasks, pedagogical decision-making, facilitation of discourse, and multiple means of assessment. Finally, the author asserts that a pre-conference between teacher and administrator is necessary so that there is a thorough understanding of what the teacher plans to have occur in the classroom. To assist administrators both in the pre-conference and during the observation, Malloy gives a list of twenty-five conference questions and things to look for in the lesson.

National Center for Educational Statistics. (2003). *Highlights from the TIMSS 1999 video study of eighth-grade mathematics teaching*. (NCES Publication # 2003–013). Washington, DC: Education Publications Center. URL: <http://nces.ed.gov/pubs2003/timssvideo/>.

How does the teaching of eighth grade mathematics differ from country to country? What are the characteristics of mathematics classrooms in high-achieving countries? What can be learned by studying teaching in other cultures? These were some of the questions that a team of researchers and videographers set out to answer in what is now known as the *1999 TIMSS Video Study*. The team visited eighth grade classrooms in Australia, the Czech Republic, Hong Kong SAR, Japan, the Netherlands, Switzerland, and the United States, taped 638 full lessons, and then analyzed and coded them to create a statistical picture of each county's mathematical content, complexity of learning tasks, pedagogy, intended vs. enacted curriculum, independent vs. group work, student-teacher dynamics, and even the number of times a lesson was interrupted. Their findings paint a portrait of how teaching is a cultural activity, with each of the seven countries having a unique "lesson signature" with its own distinctive expectations, routines, lesson timelines, etc. One particularly striking finding was that while 17% of math problems in the U.S. might be classified as having *potential* for mathematical connections, actually making those connections occurred in less than 1% of the lessons. In contrast, in high-achieving countries this occurred between 37–52% of

the time. Although the study was designed to describe, not prescribe, its findings allow U.S. educators to look at teaching practices, "because seeing one's own practices is a first step toward re-examining them, and ultimately improving them." (The online version of *Highlights*, which includes film clips from the seven participating countries can be found at: http://nces.ed.gov/pubs2003/2003011.pdf)

Nelson, B. S. & Sassi, A. (2000). Shifting approaches to supervision: The case of mathematics supervision. *Educational Administration Quarterly, 36*(4), 553–584. URL: <http://eaq.sagepub.com/cgi/content/abstract/36/4/553>.

Abstract: *Standards*-based instructional reform has been occurring in all major school subjects. However, administrators' supervisory practices have generally not taken account of subject-matter content but have focused primarily on pedagogical process. This article addresses how administrators can better support standards-based instruction by shifting their approaches to supervision to attend to the intersection of process and content. The article reports on a study that looked at what administrators thought significant when viewing the same videotape of a fifth-grade mathematics lesson at the beginning and end of a professional development seminar on supervision. It describes the different interpretations of the same events at these two times to illustrate the emergence of new principles to guide the exercise of administrators' professional judgment in classroom observation and supervision. The article concludes that there is a need to bring adequate subject-matter knowledge to the process of supervision and suggests several possible directions to achieve this shift.

Nelson, B. S., Sassi, A., & Driscoll, M. (1999). Developing a new eye for mathematical classrooms: Classroom observation and teacher supervision. *NCSM Journal of Mathematics Education Leadership, 2*(4), 4–12.

This article addresses the reshaping of administrator thinking and the role of teacher supervision in this era of mathematical reform. During the past few decades the mathematics education community has come to view learning as the process of thinking through ideas, teaching as providing the guidance and support for such thinking, and debate and discussion as integral features of mathematics classrooms that may extend, on occasion, beyond class time and classroom walls. These changes constitute a significant departure from established views of mathematics learning and teaching, and from established images of mathematics classrooms—particularly those held by administrators who supervise teachers. It was found that through professional development targeted to their supervisory role administrators were able to experience changes in several key areas of their thinking about supervision in mathematics.

Remillard, J. T., & Geist, P. K. (2002). Supporting teachers' professional learning by navigating openings in the curriculum. *Journal of Mathematics Teacher Education, 5*(1), 7–34. URL: <http://www.ingentaconnect.com/content/klu/jmte/2002/00000005/00000001/00387654>.

This article, which addresses instructional dilemmas faced by staff developers, focuses on teacher educators who were facilitating *Developing Mathematical Ideas* (DMI), a professional development curriculum designed to help elementary teachers learn about—and experience—*Standards*-based mathematics. During the teaching of DMI, course facilitators faced dilemmas when confronted with "openings" in the curriculum: unanticipated questions, challenges, or actions that required on-the-spot judgments about how to guide discourse. Three common openings were: 1) Participants seeking specific advice ("What should I do about...?"); 2) Participants offering prescriptive advice ("The only thing that works is...."); 3) The "surprise, inspiration, insight, confusion, frustration, and curiosity" that arose during the exploration of mathematical ideas. Although the course facilitators found these points tense and precarious, Remillard and Geist consider them not as something to shy away from, but as "potentially rich spaces to foster learning." They suggest that in order to navigate these awkward openings into the curriculum, facilitators need to recognize both the openings themselves and the tensions and competing goals underlying them. Then, after considering potential responses and likely consequences, facilitators will be able to take a "considered action" that allows them to navigate through the opening. While this article is grounded in the DMI curriculum, its analyses and conclusions are relevant to any staff developer working with teachers as they implement *Standards*-based curricula.

Reys, B., Chavez, O., & Reys, R. (2003). Middle school mathematics curriculum—A guide for principals. *Principal Leadership, 3*(7), 61–66.

Reys, Chavez, and Reys argue that principals can and should influence the choice of mathematics instructional materials in their schools. The article, which begins by exploring the recent status of curriculum in the U.S., asserts that principals need to understand the characteristics of a coherent mathematics curriculum and its benefits for their students. The authors explain why textbooks are crucial to what actually is taught in mathematics classrooms and make recommendations for effective instructional leadership. The article also includes two sidebar discussions, one examining the debate about whether algebra should be a stand-alone course or a strand within broader mathematics instruction, and the other advocating what mathematics middle-grade students need to know.

Sawada, D., Pilburn, M. D., Judson, E., Turley, J., Falconer, K., Benford, R., et al. (2002). Measuring reform practices in science and mathematics classrooms: The reformed teaching observation protocol. *School Science and Mathematics, 102*(6), 245–253. URL: <http://static.highbeam.com/s/schoolscienceandmathematics/october012002measuringreformpracticesinscienceandmathematicscla/>.

After having read NCTM documents, adopted a *Standards*-based curriculum, and attended professional development sessions, teachers (and their supervisors) may assume that reform-oriented teaching is underway. But what does reform-oriented teaching look like? Are there criteria for defining it? More importantly, is there an instrument for assessing it, quantifying it, and then using the resulting data to help teachers become more reform-oriented in their practice? These questions led to the design and development of the Reformed Teacher Observation Protocol (RTOP), a tool that assesses the degree to which mathematics instruction is "reformed." This particular article, which would be informative to policy makers and researchers interested in the validity and reliability of RTOP, also lists in its Appendix twenty-five factors identified as indicators of reform-oriented teaching. For professional developers, administrators, and pre-service educators there is also an introductory online video-based training available at: http://physicsed.buffalostate.edu/AZTEC/RTOP/RTOP_full. The website includes intended interpretations for each of the twenty-five descriptors, downloadable forms for RTOP data collection, and three videos created specifically for scoring practice. A model of collaborative scoring to insure reliability among observers is also described, since the authors intent is that, "The RTOP is based upon discourse, and learning to use RTOP should be a collaborative interaction between participants."

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator, 26*(2), 10–26, 47–48. URL: <http://www.aft.org/pubs-reports/american_educator/summer2002/curriculum.pdf>.

Schmidt, Houang, and Cogan examine the findings of the Third International Mathematics and Science Study (TIMSS) as it relates to curriculum and teachers' use of curriculum both internationally and in the U.S., arguing that American students deserve access to a "world-class curriculum." In their analysis of curricula in countries scoring high on the TIMSS, the authors found "a generally agreed-upon set of mathematics topics that serve as the foundation for mathematics understanding." In comparison, the curriculum in the U.S. is described as unfocused, incoherent, highly repetitive, and not very demanding. Rather than teach and reteach similar topics (e.g., operations with whole numbers) each year as is commonly done in the U.S., most high-achieving countries focus on fewer topics at each grade level and teach them for a shorter grade span. The authors point to the coherence of high-achieving countries' curricula, noting that their "intended" (national) curriculum is also the "enacted" curriculum—every child in a particular grade will be exposed to the same set of mathematical ideas during the course of the year. The authors call attention to the repetition and incoherence they perceive in U.S. mathematics curricula and suggest that these qualities have a significant impact on the performance of U.S. students on international exams. Note: The graphs in this article are especially useful in showing the contrast between the scope and sequence of curricular topics in the U.S. and abroad. Short articles on equity issues and professional development are also included within the text of the article.

Spillane, J. (2000). District leaders' perceptions of teacher learning. *CPRE Occasional Paper Series, OP–05*. Philadelphia, PA: Consortium for Policy Research in Education. URL: <http://www.cpre.org/Publications/op-05.pdf>.

This paper studies district leaders' beliefs about implementing new standards for mathematics and science in the classroom in nine Michigan school districts. The author theorizes that the way in which district leaders support teachers in implementing mathematics and science reforms will depend upon the leaders' understandings of the instructional ideas in the reforms. In analyzing the data, the author found that district leaders' perspectives fell into three categories: quasi-behaviorist, situated, and quasi-cognitive. Those leaders who viewed professional development from a quasi-behaviorist perspective believed that teachers learn best by receiving transmission of knowledge from an expert, and therefore constructed opportunities for teachers to learn about change in that manner. Teachers were taught content of the new curricular materials, new pedagogical strategies, and technology by external experts. Topics were integrated at very broad levels. Those leaders who viewed professional development from a situated perspective considered local and outside experts to be important in teacher learning as well, but accorded teachers a more active role in constructing their learning experiences. The concept of "teacher as learner" was central in the design of professional development, and the curriculum for teacher learning included students' work in the new instructional materials as well as teachers' practice implementing them. Finally, the one teacher who viewed professional development from a quasi-cognitive perspective believed that teacher learning was enabled by a teacher's own reflection on her existing knowledge, experience and practice. In thinking about professional development that was focused around the classroom curriculum, the district leader enabled teachers to appreciate their own needs as learners, using students' learning needs to understand their own. The paper closes with the author's assessment of what structures in districts constrain leaders' work to facilitate professional development. In order to improve professional development, leaders will need to challenge their theories about teacher learning.

Stein, M. K., Smith, M. S., & Silver, E. (1999). The development of professional developers: Learning to assist teachers in new settings. *Harvard Educational Review, 69*(3), 237–269. URL: <http://lsc-net.terc.edu/do.cfm/paper/8226/show/use_set-teacher_pd>.

The authors begin by contrasting forms of professional development frequently used in the U.S. with emerging ideas about how professional development could be more supportive of teachers engaged in implementing curricular reform. The article then explores two cases in which professional developers worked with middle school mathematics teachers over an extended period of time. The main goal of this work was to help teachers implement programs aligned with NCTM's *Curriculum and Evaluation Standards*. In comparing the two cases, the authors examine the challenges professional developers face when attempting to facilitate the transformation of teachers' instructional practice. The article closes by exploring the broader implications of reshaping professional development.

Stigler, J. & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12–17. URL: <http://www.pims.math.ca/education/2004/workshop/and/stigler04.html>.

In this article, Stigler and Hiebert suggest how their research findings on international differences in mathematics instruction might improve student achievement in the U.S. In their first study (Third International Mathematics and Science Study, TIMSS, 1995), which used video to observe teachers in classrooms, they discovered that there was a need for a shared language to describe teaching. They also found that even among teachers who said they had read mathematics reform documents there was "great unevenness in how teachers interpreted the reforms... and little evidence that classroom practices actually reflected the goals of the reforms." Their follow-up study (TIMSS 1999) had one particularly striking finding: U.S. mathematics teachers turned most problems into procedural exercises—or actually gave students the answers. "Therefore," the authors state, "the lower achievement of U.S. students cannot be explained by an overemphasis on concepts and understanding. In fact, U.S. 8th graders spend most of their time practicing procedures. They rarely spend time engaged in the serious study of mathematical concepts." The authors share three broad ideas for improving the teaching of mathematics in the U.S.: 1) Focus on the cultural activity of teaching, including the way teachers and students interact with and connect mathematical ideas; 2) Analyze teacher practices and their effects on student learning; 3) Build a shared knowledge base of research and alternative images of teaching.

Implementation

Selecting a Standards-*based mathematics curriculum that is right for your district takes time and involves careful attention to criteria important to your stakeholders. Choosing a curriculum, however, is only the beginning. It is equally important to be aware of challenges and considerations that accompany your curriculum's implementation. The articles in this section share the thoughts and experiences of teachers, students, parents, administrators, and researchers—all of whom have been affected by the implementation of* Standards-*based curricula.*

The first group of articles addresses factors that often act as barriers to successful implementation (e.g., issues of change, teacher support, and beliefs about how children learn mathematics) along with strategies for overcoming these challenges. The remaining sections deal with specific questions asked by those making and affected by curriculum decisions. For example, teachers and administrators often worry about parents' reactions to a curriculum different from what they experienced as students. The articles in the "Working with Parents" section highlight examples of districts that have raised awareness of Standards-*based curricula in order to help parents understand what their children are learning and why.*

Challenges and Strategies

Bay, J. (2000). *The dynamics of implementing and sustaining* Standards-*based mathematics curricula in middle schools.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document #ED 441676).

Based on case studies conducted in two districts that had recently implemented *Standards*-based mathematics curricula at the middle school level, this paper explores district- and teacher-level changes resulting from the implementation. In particular, the study examines two questions: 1) What district-level constraints and considerations impact decisions regarding implementation of a *Standards*-based curriculum?; 2) How do individual teachers respond to the decision to implement a *Standards*-based program? In response to the latter question, the article explores whether teachers' implementation was faithful to the program's philosophy, and why and in what ways their instruction changed as a result of their level of implementation. The study also explores the impact of parents' expectations and of teachers' comfort level with the implementation.

Bay, J., Reys, B., & Reys, R. (1999). The top 10 elements that must be in place to implement *Standards*-based curricula. *Phi Delta Kappan, 80*(7), 503–506.

This article maintains that several common factors contribute to the effectiveness of teachers in implementing a *Standards*-based mathematics curriculum in their classrooms. Awareness of these factors and the development of ways to address them will increase the likelihood of success when implementing any new and different curriculum. A new curriculum demands changes in instruction, content, assessment, and homework, and thus requires hundreds of hours for change to occur. This article is based on three years of working with middle-school mathematics teachers from 23 school districts in the Missouri Middle-school Mathematics Project, funded by the National Science Foundation.

Bay-Williams, J., Reys, B., & Reys, R. (2003). Effectively implementing *Standards*-based curricula in middle schools. *Middle School Journal, 34*(4), 36–41.

The article opens with an explanation of what *Standards*-based curricula are and the challenges they pose for student and teachers. The principles of effective implementation shared in the article are gleaned from 23 Missouri teachers who piloted Standards-based materials during a teacher enhancement project. Their suggestions include: building support among local administrators and community members; helping teachers and parents understand the scope, content, and approach of the new materials; and creating long-term support for the implementation. The authors recommend that schools utilize recommendations of those who have gone through an implementation, and learn from their mistakes.

Bell, M. (1998-1999). Problems with implementing new curricula: The example of the K–6 *Everyday Mathematics* curriculum. *UCSMP Newsletter, 24*, 1–2.

This article is a talk delivered to the Fourth International UCSMP Conference on Mathematics Education, August 5–7, 1998, by UCSMP Elementary Materials Director Max Bell. Bell addressed some dilemmas that face *Standards*-based curricula as a whole, including: the tradition of "basics" in school mathematics content, "objective" grading in terms of tests and report cards, the use of "old" tests to measure new emphases in mathematics instruction, the development of concepts over time compared to testable mastery of skills, the various false dichotomies that are set up in many reform or anti-reform arguments (objective versus alternative assessment, use or non-use of calculators, teaching directly versus

collaborative learning), reflex conservatism and defensiveness, global adoptions of curricula, and the expense of implementing a new curriculum. In conclusion, Bell provides a preliminary "Where to go from here" outline for curriculum developers that recommends: curricular revisions, better professional development, technology toolkits, more implementation options, and on-line technical support for users of the curriculum.

Briars, D. (1999). Square one: Promoting systemic math reform. *The School Administrator, 56*(1), 39–43. URL: <http://www.aasa.org/publications/sa/1999_01/Briars.htm>.

Briars addresses systemic mathematics reforms with particular attention to the following eight issues: 1) High stakes assessments must be tied to appropriate instructional targets; 2) *Standards*-based instruction is more than using manipulatives and cooperative learning; 3) Teachers need *Standards*-based instructional materials; 4) Teachers need substantial, continuing professional development and in-class support; 5) Administrators must recognize and support *Standards*-based instruction; 6) District policies and practices will have to change; 7) Materials and services are available to support mathematics reform; 8) Reform has a payoff for all students. In addition, Briars includes a short piece on educating parents about mathematics education reform, stressing that parents play an important role in reform efforts.

Orrill, C. H., & Anthony, H. G. (2003). *Implementing Reform Curriculum: A Case of Who's in Charge*. Paper presented at the American Educational Research Association's Annual Meeting, Chicago, IL. URL: <http://www.orrill.com/chandra/barriers.pdf>.

Implementing curricula aligned with reform documents such as NCTM's *Principles and Standards* often challenges teachers' beliefs and pushes for a profound change in the way they teach and understand mathematics. In this article, Orrill and Anthony provide a deeper look at what is happening in rural schools where six middle and high school mathematics teachers are implementing two reform curricula whose theoretical base is much different than what the teachers have previously experienced. Specifically, the authors identify and unpack the barriers teachers faced during the implementation and the factors that shaped their interpretations of these barriers. Orrill and Anthony found that teachers felt the reform materials called for very different pedagogical approaches, ones that pushed their comfort levels and their mathematical knowledge, and which strongly influenced their view of the materials. The barriers (e.g., concerns about student skills, external expectations) that teachers experienced and identified were affected by their beliefs about mathematics teaching and learning as well as other perceptions (e.g., teacher confidence, teachers' definition of success), and each of these elements influenced the success of their implementation. The authors describe how teachers addressed the barriers and how this played out in the implementation of the curriculum. In conclusion, they recommend that during the adoption and implementation process it is necessary to consider the influences at play, which barriers are controllable at the local level, and how to address the deeper issues and true barriers that accompany implementation.

Reys, R., Reys, B., Barnes, D., Beem, J., & Papick, I. (1998). What is standing in the way of middle school mathematics curriculum reform? *Middle School Journal, 30*(2), 42–48.

The authors of this article relate their experiences observing the implementation of several NSF-funded, NCTM *Standards*-based, middle grades mathematics curricula. The data collected by the authors is made up of teacher and student journals, reflections, e-mails, and notes from meetings, group discussions, and conversations. Major obstacles to curriculum reform are revealed to be the following: traditional beliefs and practices regarding school mathematics; initial (negative) student reactions to new curricula and teaching practices; parental perceptions and concerns; the transition from the middle grades to high school; assessment (measuring and monitoring) of student learning; teacher preparation and professional development; and support from administrators and teacher colleagues. In conclusion, the article supports the need for national change in mathematics education. The authors encourage teachers and administrators to "weather the occasional storms," and advocate the need for real support for teachers as they move to make changes in their classrooms.

Russell, S. J. (1998). Mathematics curriculum implementation: Not a beginning, not an end. *Hands On!, 21*(1), 6–9, 29. URL: <http://www.terc.edu/handson/s98/russell.html>.

The author maintains that systemic reform in mathematics requires a multi-year commitment that includes at least three intertwined substantive components: staff development, curriculum implementation, and community education. This article examines the role curriculum can play in systemic reform and then provides examples of school systems that have taken seriously the commitment and allocation of resources that are necessary to engage in systemic change.

Spillane, J., Reiser, B., & Reimer, T. (2002). Policy implementation and cognition: Reframing and refocusing implementation research. *Review of Educational Research, 72*(3), 387–431.

While this article does not speak specifically to mathematics reform, it offers a detailed perspective of why reform, change, and implementation are so difficult, providing insight for mathematics educators who regularly face policy initiatives developed at the state and national level. The article examines

how education policies are carried out at the local level and how local implementing agents act as individual sense-makers. The authors studied how local agents receive and interpret information as well as how "prior knowledge, beliefs, and experiences influence the construction of new understandings." The researchers suggest that most theories about why policy implementation does not work "fail to take account of the complexity of human sense-making." They then go on to describe a model for how individuals tend to understand a policy message, how they interpret the message with regard to their own practice, and what potential changes they imagine in their practice as a result. An implication of this model is that "different agents will construct different understandings based on their prior knowledge and different interpretations of the same message." The authors also speak to certain obstacles that often arise during the implementation of new policies. For example, one might reject information that does not match current beliefs and expectations. New policies often require a change in existing knowledge, and making this change can be difficult. Additionally, people often see new ideas as familiar, either because they attend only to the familiar or because they treat the ideas superficially and miss the deeper meaning. Although these obstacles are discussed in general, they often arise specifically within the context of implementing mathematics reform curricula. A key message of the authors is that "it is not enough simply to communicate the policy. There is a critical need to structure learning opportunities so that stakeholders can construct an interpretation of the policy and its implications for their own behavior."

Thompson, D. R. & Kersaint, G. (2002). Impediments to the implementation of a successful middle-grades mathematics program: Implications for mathematics. *NCSM Journal of Mathematics Education Leadership, (6)*1, 21–27.

Thompson and Kersaint describe how a mix of teachers, mathematics supervisors, and administrators came to consensus on ten barriers (e.g., an unfocused curriculum, time, lack of professional development opportunities, insufficient time for planning) that would impede effective curricular implementation. Once these factors were identified the team developed solutions either to prevent them or to address them if and when they occurred. Most importantly, they specified at what level responsibility for dealing with each impediment lay. While this article focuses on the mathematics supervisor's responsibility in an effective implementation plan, the full report (available at http://www.coedu.usf.edu/fjer/2001/FJERV41P4378.pdf) includes recommendations for other members of the implementation team. The article would also be helpful for districts that are revisiting their implementation plan because of the impediments they have encountered.

Equity

Baxter, J., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal, 101*(5), 529–547.

In this study, conducted over a full school year, researchers studied the classroom dynamics of reform-based mathematics instruction, paying special attention to the participation of 16 low-achieving students in five *Everyday Mathematics* classrooms. Student involvement in whole-class discussions and partnership work was observed, and their teachers were interviewed. Although the researchers saw a relatively low level of participation by these students overall, they found positive differences in classrooms where the teachers used specific strategies to increase these students' participation. These findings led the authors to conclude that in order for low-achieving students to succeed in reform-based mathematics programs, teachers and administrators must provide instructional and structural supports to encourage these students' active participation.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239–258. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2002-07-239a&from=B>.

While some researchers have expressed doubts that reform-oriented curricula can promote equity, the author of this article believes differently. "Although it is very important to realize that some students may be less prepared than others to engage in the different roles that are required by open curricula, analyses that go from this idea to the claim that traditional curricula are more suitable may be very misleading." The author contends that a shift needs to occur, away from "what students *cannot* do" to "what schools *can* do," so schools do not reinforce existing social class disparities. Boaler cites research showing it is not just curriculum materials alone that need to be considered when addressing questions of equity, but the *enacted* curriculum, the manner in which teachers present the materials to students, as this may differ from what was intended by curriculum developers. Data are presented from two studies in which middle and high school teachers using reform-oriented mathematics curricula achieved a reduction in linguistic, ethnic, and class inequalities in their schools. The teaching practices that these teachers employed, 1) introducing activities through discussion, 2) teaching students to explain and justify, and 3) making real world contexts accessible, were central to the attainment of equality. These findings suggest that it is crucial to go beyond the curriculum and to consider

the impact of teachers and their instructional practices when developing or evaluating programs designed to promote equity.

Bussey, J. (2001). Mathematics for the alternative high school student. *The Journal of Court, Community, and Alternative Schools, 14*, 45–51. URL: <http://www.mathimp.org/downloads/research/J_Bussey_Article.pdf>.

This article presents the *Interactive Mathematics Program* (IMP) as an ideal curricular option for alternative high schools whose students have generally been unsuccessful or turned off to mathematics. The author notes the success of IMP in these types of schools and identifies five reasons for the effectiveness of the program. The *Interactive Mathematics Program* 1) recognizes diverse learning styles, 2) uses open-ended problems and explorations, 3) employs real-world context, humor, and emotion, 4) presents mathematical ideas useful to the average adult, and 5) uses a variety of assessment tools. In concluding, the author notes some of the challenges in implementing the curriculum in an alternative school and emphasizes the importance of professional development.

Garfunkel, S. & Froelich, G. (1999). Helping students see the world mathematically. In L. J. Sheffield (Ed.), *Developing Mathematically Promising Students*. (pp. 154–184). Reston, VA: National Council of Teachers of Mathematics.

In order to engage students who are mathematically promising, mathematics classes need to attract and challenge students by helping them see the importance of mathematics in their world. The authors of this article suggest that good mathematics problems are engaging, accessible at different levels for students with varying degrees of mathematical experience, and extendible. The article cites several sources for such problems, including National Science Foundation-funded curriculum projects that have student engagement as one of their goals. In closing, the authors suggest that not only are engaging problems good tools for mathematically promising students, but that problems such as these can actually help teachers identify students who have potential for looking at the world mathematically.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2003-01-37a&from=B>.

The author conducted a two-year study about teaching and learning mathematics for social justice in his middle-school, urban, Latino classroom. While his instruction was based in the *Mathematics in Context* curriculum for grades 7 and 8, Gutstein also created some projects that were specifically relevant to the lives of the students in his classroom. He explains, "Using qualitative, practitioner-research methodology, I learned that students began to *read the world* (understand complex issues involving justice and equity) using mathematics, to develop mathematical power, and to change their orientation toward mathematics." Additionally, Gutstein poses the hypothesis that several specific conditions must exist for students to learn mathematics for social justice: the activities of the class must examine complicated issues of personal importance to students; the classroom culture must support inquiry into and analysis of these real-world contexts; and the activities of the classroom must be coherent with the features of the curriculum materials used. Finally, since his students were accustomed to applying challenging mathematical ideas to "real world" context from their use of the *Mathematics in Context* curriculum, Gutstein argues that he felt comfortable taking the freedom to design additional projects that were specifically relevant to his students' lives.

Hirsch, C. & Weinhold, M. (1999). Everybody counts—Including the mathematically promising. In L. Jensen Sheffield (Ed.), *Developing Mathematically Promising Students* (pp. 233–241). Reston, VA: National Council of Teachers of Mathematics.

The authors, one of whom is a curriculum developer and the other a classroom teacher, explain that although *Standards*-based mathematics curricula were designed for use in heterogeneous classrooms, they have flexibility to accommodate the needs of mathematically promising students. In describing the design components of *Contemporary Mathematics in Context* (Core-Plus) Hirsch and Weinhold illustrate how this particular curriculum has the flexibility to meet the needs of students from a range of abilities. The article also stresses that an important component in challenging mathematically promising students in heterogeneous classrooms is the teacher's ability to adapt instruction to a range of students' needs.

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*(4), 454–482. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=JRME2000-07-454a&from=B>.

Lubienski explores the experiences of students of different socioeconomic status (SES) in a 7th grade mathematics class piloting materials from the *Connected Mathematics Project* (CMP). (Students in the study had used CMP materials during the year preceding the study, as well.) Lubienski noticed trends in students of different SES in their response to several aspects of the curriculum: methods of instruction used with CMP materials, the open nature of the problems, and the contextualization of the problems. The article discusses factors that may have led to these SES differences, but discourages premature conclusions regarding the response of students from different SES levels to reform curricula.

Perez, C. (2000). Equity in the *Standards*-based elementary mathematics classroom. *ENC Focus, 7*(4), 29–31. URL: <http://www.terc.edu/wge/equity.html>.

Perez argues that disparities in children's early education affect their mathematics choices throughout their lives. This article acknowledges that while gaps in mathematical achievement have lessened as a result of reform initiatives such as those embodied in NCTM's *Principles and Standards*, inequities still remain. Highlighting methodologies and strategies from the *Standards*, the author emphasizes the teacher's role as a facilitator for all students' learning. In conclusion, the author recommends strategies for creating a more equitable learning environment for teachers who are using cooperative grouping and inquiry-based learning.

Tevebaugh, T. (1998). Mathematics is not a universal language. *Teaching Children Mathematics, 5*(4), 214–215.

This opinion article presents the difficulty that ESL (English as a Second Language) and LEP (Limited English Proficiency) students have with mathematics class, in part because of the complex vocabulary such classes require. Tevebaugh talks about the frustration of many language-minority students who feel that their needs are not being met in today's mathematics classroom, especially with the NCTM *Standards*' call for more language-based learning, teaching, and assessment. Tevebaugh offers strategies for teachers of language-minority students: 1) Create a safe and trusting environment; 2) Adjust teaching methods to make lessons more comprehensible to LEP students by using visual aids, targeting vocabulary, and becoming aware of language problem areas; 3) Use multicultural teaching materials to show respect and openness to other cultures in the classroom. Also cited are sources to find other strategies for helping LEP students adjust to a *Standards*-based mathematics classroom.

Woodward, J., & Montague, M. (2002). Meeting the challenge of mathematics reform for students with LD. *The Journal of Special Education, 36*(2), 89–101. URL: <http://static.highbeam.com/j/journalofspecialeducation/june222002/meetingthechallengeofmathematicsreformforstudentsw/>.

Special educators have had longstanding concerns about how their students could meet the high standards proposed by NCTM's reform documents. Many felt the *Principles and Standards* failed to specify how curricular and pedagogical reforms should be implemented in their classrooms and questioned what the *Standards*' effect should be for students with learning disabilities. According to this article, two main criticisms are 1) that the shift away from traditional teaching methods (i.e. direct instruction) toward constructivism is problematic for students with learning disabilities, and 2) that the research base of the NCTM *Standards* is insufficient to help special educators meet the *Standards* in an appropriate manner. Although research in mathematics education, especially reform mathematics, for students with learning disabilities is still sparse, the authors point to directions that might hold promise for improving the performance of these students. They cite studies and strategies used for mathematics instruction with learning disabled students and attempt to synthesize the state of special education within mathematics reform. They propose that resources need to be increased in order to enable changes in the student experience as suggested by the *Standards*. Special education students also have limits on their time due to their needs beyond the mathematics classroom, and the authors ask, "What is the most worthwhile use of limited instructional time for these students?" Citing Hiebert (1999), they suggest, "The implications of mathematics reform for special education should be grounded in what we value educationally."

Assessment

Billstein, R. (1998). The STEM model. *Mathematics Teaching in the Middle School, 3*(4), 282–296.

This article describes the STEM (*Six Through Eight Mathematics*) project's assessment package, designed to "raise the level of students' performance in problem-solving and communication along with performance in mathematical content." Billstein focuses on STEM's (now called *MATH Thematics*) developed assessment package and its use of open-ended questions and real-world applications. The assessment package contains student self-assessment and teacher assessment criteria, and the article demonstrates the use of both tools with an example of student work. Feedback on these methods reflect that it takes some time for both students and teachers to fully learn and be comfortable with the STEM package, but that it gives students and teachers new vocabulary for communicating about their problem-solving methodology.

Bright, G. W., & Joyner, J. M. (2004-2005). Classroom assessment in middle grades and high school. *NCSM Journal of Mathematics Education Leadership, 7*(2), 11–17. URL: <http://www.fi.uu.nl/catch/documents/AssessmentPaper1999.doc>.

Reform mathematics not only has implications for the way mathematics is taught and learned, but also for the way mathematics is assessed. Formal assessments are no longer sufficient as a sole source for understanding what students know about mathematics, and it is the informal practice of classroom assessment that Bright and Joyner discuss in this article. They argue that despite changes in assessment practices, few opportunities exist for professional development where teachers can better understand how to collect information about students' thinking and use this data to design instruction.

Keeping this in mind, they describe Dynamic Classroom Assessment (DCA), a professional development program supported by an NSF grant. DCA was designed to help middle and high school teachers incorporate classroom assessment into their instructional planning. DCA aims to help teachers 1) set learning goals and evaluate how different assessment methods align, 2) differentiate between errors in what students know versus errors in what students show they know, 3) use questioning in a skillful way, and 4) reflect on and improve instructional decision-making. The authors share a vignette of one teacher's growth, as she participated in DCA. Bright and Joyner conclude that using classroom assessment helps students learn more and teachers learn more about their students' thinking.

Burrill, J., Feijs, E., Meyer, M., van Reeuwijk, M., Webb, D., & Wijers, M. (2001). *The role of assessment in* Standards-*based middle school mathematics curriculum materials*. St. Louis, MO: Show-Me Center. URL: <http://showmecenter.missouri.edu/resources/assessment.pdf>.

This brochure is designed as a resource on assessment in five *Standards*-based middle-grades mathematics curricula: *Connected Mathematics*, *Mathematics in Context*, *MathScape*, *MATH Thematics*, and *Middle School Mathematics through Applications*. It also provides information pertinent to users of *Standards*-based programs at other grade levels. The document details the role of classroom assessment when using *Standards*-based materials, and includes samples of assessment tasks as well as expectations for both teachers and students. The brochure also addresses external assessments, providing information about a range of alternatives for external assessment.

Cole, K. (1999). Walking around: Getting more from informal assessment. *Mathematics Teaching in the Middle School 4*(4), 224–227. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-01-224a&from=B>.

Cole describes "Walk-Around Assessment," an important evaluation strategy used by teachers working with materials from the *Middle School Mathematics Through Applications Project* (MMAP). He notes that focused walking-around helps make more efficient use of time spent assessing and "adds immeasurably to the sense of keeping up with students' varied progress and needs." The technique features observations, on-the-spot assessment charts, and conferences. Cole shares ideas on how to take notes, how to create continuity among informal assessments, and how to walk around equitably while managing one's classroom. Finally, Cole addresses how to integrate this type of informal assessment with more traditional quizzes, tests, and assignments.

Cole, K., Coffey, J., & Goldman, S. (1999). Using assessments to improve equity in mathematics. *Educational Leadership, 56*(6), 56–58.

This article emphasizes the importance of the student role in classroom assessment and suggests a strategy of asking students to consider four key questions when exploring mathematical concepts: 1) What are we learning? 2) What is quality work? 3) To whom do we hold ourselves accountable? 4) How do we use assessment tools to learn more? Included in the article are examples of how students used this assessment strategy, which was developed through field tests conducted on the *Middle School Mathematics through Applications Project* (MMAP). The authors argue that assessment can be a powerful tool and, when used effectively, can make mathematics accessible to all learners.

Kantrov, I. (2000). *Assessing students' mathematics learning*. K–12 Mathematics Curriculum Center Issues Paper Series. Newton, MA: Education Development Center, Inc.

This paper explains the different kinds of tests used to assess mathematics learning. It also can help readers answer questions about the compatibility of student assessments with the content and approaches to teaching that are embodied in *Standards*-based curricula. More specifically, the paper addresses 1) terms used in debates about mathematics assessment, 2) the advantages and disadvantages of different kinds of assessments when used for different purposes, 3) evidence regarding the impact of new mathematics curricula on student achievement on high-stakes tests, and 4) criteria to apply to your own school or district assessments.

Kitchen, R., Cherrington, A., Gates, J., Hitchings, J., Majka, M., Merk, M., et al. (2002). Supporting reform through performance assessment. *Mathematics Teaching in the Middle School, 8*(1), 24–30. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2002-09-24a&from=B>.

This article describes the efforts of a group of middle school teachers to align assessment tasks with their reform mathematics curriculum. Together they created a series of performance assessment tasks for sixth, seventh, and eighth grade units. (Samples of the tasks and their scoring rubrics are included in the article.) These tasks measured key concepts taught, pushed students to apply their knowledge in real-life contexts, and encouraged them to use higher-order thinking. The authors share multiple benefits that they experienced as a result of designing and using the performance tasks. First, teachers benefited from collaboration with one another. They became more focused on what mathematics they were teaching and more connected to students in other teachers' classes. They acquired a stronger understanding of student misconceptions as a result of scoring and discussing the performance assessments together, thereby improving their teaching as a result of these analyses. They also felt that this project

contributed to their school's larger reform efforts. Eliminating tracking had been critical to their success and despite early concerns, parents reported being pleased with the mathematics education their children received. Finally, the teachers found that students took the performance assessments seriously and worked hard to succeed.

Kouba, V. (1999). Multiple interpretations = more challenges. *Mathematics Teaching in the Middle School, 5*(4), 232–235. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS1999-12-232a&from=B>.

Kouba addresses the way teachers are confronted with new pedagogical and assessment challenges as they use curricular activities in which the mathematics is embedded in contexts. Figuring out the reasonableness of an answer is a complex process; teachers working with curricula that present problems in contexts find they need to look closely at students' explanations to determine students' understandings of the mathematics concepts. Several examples in the article illustrate the ways students and teachers interpret problems differently. The author explores teachers' varying assessments of student responses and closes with lessons learned, as well as tips for teachers about assessing mathematics taught contextually.

Moskal, B. (2000). Understanding student responses to open-ended tasks. *Mathematics Teaching in the Middle School, 5*(8), 500–505. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2000-04-500a&from=B>.

Assessing students' understanding based on their responses to open-ended tasks, such as those found in *Standards*-based mathematics curricula, requires teachers to develop new skills. This article explores four student responses to a problem about the area of an irregular shape, then includes the teacher's reaction to each solution. These examples illustrate ways in which the teacher learned to look differently at students' work in order to assess for understanding.

Wilson, L. D., & Kenney, P. A. (2003). Classroom and large-scale assessment. In J. Kilpatrick, G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 53–67). Reston, VA: National Council of Teachers of Mathematics.

In this overview article, Wilson and Kenney discuss assessment in relation to mathematics reform by using the six assessment standards laid out in the *Assessment Standards for School Mathematics*. They highlight the impact of assessment for teachers, students, parents, administrators, and policy makers. In this article, the authors define assessment to incorporate "all of the activities that educators (or students themselves) use to learn what students know and can do in mathematics." They differentiate between classroom assessment (formative tasks that help teachers make decisions about instruction and track students' progress) and large-scale assessment (e.g., standardized tests such as the Iowa Test of Basic Skills, which are summative in nature). The authors describe features that affect assessment (e.g., teachers' knowledge), characteristics that are necessary to consider when designing assessment tasks (e.g., reflecting on the mathematics students need to know and promoting equity), and inferences that can be made from assessment tasks.

Technology

Cuoco, A., Goldenberg, E. P., & Mark, J. (1995). Technology and the mathematics curriculum: Some new initiatives. *Mathematics Teacher, 88*(3), 236–240.

This article presents responses from three comprehensive high-school level *Standards*-based curricula (*Interactive Mathematics Program*, *SIMMS Integrated Mathematics*, and the UCSMP secondary curriculum) and from one geometry curriculum (*Connected Geometry*) to the NSF technology initiatives and the 1989 NCTM Standards' recommendation that calculators and at least one computer be available for students in today's mathematics classrooms. *Interactive Mathematics Program* (IMP), discusses the role of the graphing calculator as the primary technological tool in its secondary curriculum, using problems from the curriculum to show how calculators might be used by students in IMP classes. *SIMMS Integrated Mathematics* classrooms are ideally equipped with a graphing calculator for each student, at least one computer for every four students, an overhead projector, an overhead computer projector, and on-line capabilities; computers in these classrooms should have a spreadsheet program, a geometry drawing package, a graphing package, a word processor, a symbolic manipulator, and a data-analysis program. UCSMP requires scientific calculators for all courses, assumes the use of graphing calcuators for tenth, eleventh, and twelfth graders, and assumes a statistics package for use in tenth grade. UCSMP has developed three software packages to support technology use. *Connected Geometry* advocates the use of *Geometer's Sketchpad*, *Cabri II*, or the *Geometry Inventor*, and has developed a CD-ROM for use in lesson preparation.

Draznin, S. (1997). Helping students and parents understand the important role of calculators. *TeacherLink Newsletter, 5*(2).

This short article addresses the importance of calculator use in elementary school mathematics curricula such as *Everyday Mathematics*, and suggests ways to answer parents' questions about calculator use. Sheila Sconiers, director of the Teacher Development component of UCSMP, has studied student calculator use in the primary grades and concludes that after

using calculators for a full year, students performed as well on tests of computation as did their non-calculator using peers. In recent years, resistance to using the calculator in primary grades stems from a belief that if students use calculators, they will become dependent upon them and not learn paper-and-pencil calculation skills. Draznin sees the benefits of using calculators appropriately to strengthen mathematical thinking skills; students can use their calculators to practice and experiment with concepts such as number order and magnitude. Students see the calculator as a tool for working with large numbers beyond their level of computational skill.

Goldenberg, E. P. (2000). *Thinking (and talking) about technology in math classrooms*. K–12 Mathematics Curriculum Center Issues Paper Series. Newton, MA: Education Development Center, Inc.

One of the strongest forces in the contemporary growth and evolution of mathematics and mathematics teaching is the power of new technologies. Choosing educationally appropriate technology for a classroom, however, requires sound judgment based on specific criteria. This article addresses both beneficial effects of technology as well as problems it can pose for teaching and student learning. It also discusses what educators should consider when selecting appropriate technology to support mathematics instruction.

Hillman, S. L., & Malotka, C. M. (2004). Changing views: Fearless families conquering technology together. *Mathematics Teaching in the Middle School, 10*(4), 169–173. URL: <http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2004-11-169a&from=B>.

In this article, Hillman and Malotka describe how they addressed middle school parents' commonly held misconceptions about the role of technology in the classroom. The authors offered three workshop sessions with the goal of helping parents understand the appropriate role of technology, support and help their children with mathematics, connect the use of technology to potential improvements in standardized testing, develop comfort in using fraction and graphing calculators, and build excitement together with their child. The authors also hoped that as parents explored the use of technology in their children's classrooms, parental support for reform mathematics would increase. During the first session parents had the opportunity to discuss national and state standards in mathematics and technology, as well as the role of technology in standardized testing. In addition, parents had the chance to explore the calculators before their children joined them in sessions two and three. Both of these sessions allowed parents and their children the opportunity to explore calculator activities together and engage in mathematics lessons in which technology played an integral role. These lessons were facilitated by the middle school mathematics teachers and included lessons from *Connected Mathematics*. Both parents and students recognized how appropriately used technology could help develop and illustrate key mathematics concepts and facilitate their understanding. Parents who had initially felt using the calculator was in some way cheating now viewed technology as a tool to enhance their child's mathematical knowledge.

Working with Parents

Anhalt, C. O., Allexsaht-Snider, M., & Civil, M. (2002). Middle school mathematics classrooms: A place for Latina parents' involvement. *Journal of Latinos and Education, 1*(4), 255–262. URL: <http://www.leaonline.com/doi/abs/10.1207/S1532771XJLE0104_5>.

This article suggests that the mathematics education community needs to develop and research innovative ways in which parents can be "invited to participate in examining and improving mathematics education." The example presented here, of classroom observations by parents, suggests one such promising strategy. To better understand Latina parents' views of their children's experiences in a mathematics classroom, the authors invited three Latina parents of middle school students to observe and participate in three different classrooms in which reform mathematics was being taught. A facilitator met with the parents prior to the observations to explain the objectives of the lesson and to discuss any questions, concerns, and expectations parents had about it. After the observations, the parents asked questions of the teachers and were debriefed by the facilitator. As a result of being able to observe students and teachers in the classrooms, parents became more aware of new approaches and strategies for teaching and learning mathematics. These findings suggest that parents who are provided opportunities to understand reform curricula more deeply will be better able to support their children's learning of mathematics through such curriculum materials.

Bay-Williams, J. & Meyer, M. (2003). Help! Parents and *Standards*-based mathematics. *Principal Leadership, 3*(7), 54–60.

Bay-Williams and Meyer outline questions and concerns of parents that often accompany the adoption and implementation of a *Standards*-based curriculum. They then emphasize the need for principals to understand the tenets of reform in order to respond to the issues raised by parents. According to the authors, parent queries often deal with pedagogy (e.g., the teacher's role), content (e.g., the learning of basic skills), evidence (e.g., data showing increased test scores), and questions that are curriculum specific (e.g., transitions to a different curriculum). The article provides principals with recommendations: hosting a parent math night, documenting effectiveness of the curriculum, and describing specific strategies associated with *Standards*-based classrooms,

such as heterogeneous grouping and cooperative learning. The authors believe that administrators using these suggestions will be helping parents understand why, how, and what their children are learning, and what *Standards*-based mathematics is all about.

Hendrickson, S., Siebert, D., Smith, S., Kunzler, H., & Christensen, S. (2004). Addressing parents' concerns about mathematics reform. *Teaching Children Mathematics, 11*(1), 18–23. URL: <http://www.findarticles.com/p/articles/mi_hb3451/is_200408/ai_n8218725>.

After working with hundreds of parents during their district's reform mathematics curricular implementation, the authors (K–12 mathematics teachers, mathematics specialists, and researchers) found that two types of settings have been particularly effective in addressing parent concerns: evening meetings with groups of parents, and informal one-to-one conversations that addressed common concerns. The purpose of the evening group meeting was to introduce the curriculum, explain the purpose for change, and answer participant questions. The meeting began with a general presentation where commonly anticipated concerns were addressed and parents could work on a mathematics problem. Print material providing tips on helping with homework was handed out. This general session was followed by either classroom visits for parents who seemed satisfied by the information they had received or a question-and-answer session with school and district representatives for the smaller group of parents who were more vocal about curricular change. The one-on-one conversations allowed teachers to talk to parents about what is important to know and be able to do in mathematics, the way students will be taught, and how students will be prepared for higher mathematics, college, and the world of work. An important component in these discussions was sharing with parents what educators value in children's mathematics understanding, and suggesting to parents how they could encourage those values despite the differences in how they may have learned mathematics when they were in school. Through this two-pronged approach the authors have been able to gain parents' trust, show them the complexity involved in learning mathematics, prepare them to help their children with homework, and share with them the power of the mathematics that their children will be learning.

Lubienski, S. T. (2002). *Traditional or problem-centered mathematics? The choices of students and parents in one district*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA. URL: <http://www.public.iastate.edu/~stl/aeraames2002.pdf>.

In this article, Lubienski describes one district's experiences when they offered students a choice between a traditional algebra sequence and an integrated sequence using *Contemporary Mathematics in Context* (Core-Plus). Although students had used *Standards*-based materials throughout elementary and middle school, over 80% chose the traditional sequence. Lubienski interviewed both students and their parents to determine what factors influenced their decisions. She found that parents choosing the traditional algebra sequence were most concerned about their children's academic preparedness. Choice was also correlated with socioeconomic status (SES), with students from lower SES more often enrolling in the traditional algebra sequence. Lubienski found that while gender did not predict any strong patterns, more males than females chose the integrated sequence and reported wanting to be challenged in mathematics classes. The data also highlighted the potential of increasing instructional differences and promoting inequalities based on gender, SES, and race. Although these findings clearly articulate the challenge of implementing reform in the face of anti-reform beliefs, the author hopes that those making curricular decisions continue to advocate for *Standards*-based instruction while trying to understand and address parents' concerns.

Meyer, M., Delagardelle, M. & Middleton, J. (1996). Addressing parents' concerns over curriculum reform. *Educational Leadership, 53*(7), 54–57.

This article discusses concerns that some parents feel over the new methods and mathematics of *Standards*-based curricula, and how an Iowa school district field-testing *Mathematics in Context* (MiC) addressed those concerns. The article briefly reviews the changes the curriculum brought to the Ames, Iowa mathematics classrooms, and five kinds of parent concerns (both supportive of the curriculum and not) that emerged from those changes. These concerns were: parents who back both the program and reform often requested more teacher support than the district could provide; misinformation or no information about the program's philosophy and content; the program's implementation and what to expect from students' and teachers' progress over time; lack of trust in the teachers and fears that their children were involved in an educational experiment; and traditional beliefs about schooling and commitment to memorization and repetition. The implementation group found that the last concerns were the most difficult to quell. Some "common sense strategies" used by the school district were: confidence, treating parents as equal partners, honesty, defining accountability, communication, selecting support people carefully, doing mathematics with parents, organizing family math nights, and developing family involvement packages. The district anticipated these concerns, addressed them early, and made parents allies in MiC implementation.

Peressini, D. (1997). Parental involvement in the reform of mathematics education. *Mathematics Teacher, 90*(6), 421–427.

This article addresses relationships between high school mathematics teachers and parents by looking at their different

roles in reform efforts at eight high schools and one laboratory school observed by the National Center for Research in Mathematical Sciences Education (NCRMSE). The article reviews interview data that reveal parental perspectives on the following reform elements: student comments about classroom activities, making sense of new-looking mathematics content, involvement in and difficulty with home activities, and uncertain outcomes of new methods of teaching. Interpretations of these data were based on the assumption that parents learned mathematics in a more traditional "drill and practice" setting. Teacher and school responses to the following elements are also reviewed: communication with parents, community collaboration with local businesses, observations of parent visits, parental choice and alternatives to reform, and parental resistance. Peressini concludes that parents become an essential piece of reform in schools that recognize the importance of parental support, respond to parents, and involve them in their children's education.

Peressini, D. (1998). What's all the fuss about involving parents in mathematics education? *Teaching Children Mathematics, 4*(6), 320. URL: <http://www.findarticles.com/p/articles/mi_hb3451/is_199802/ai_n8219350>.

As the political climate moves to involve parents more actively in their children's mathematics education, Peressini points to research that supports parent involvement. In this article, he refers to six categories in Epstein's typology of parental involvement: parenting, communicating, volunteering, learning at home, community collaboration, and decision-making. Peressini maps onto each of these categories ways that schools can encourage and utilize parent participation in their children's mathematics education, in the selection and implementation of mathematics curricula, and in the involvement of the larger community in the school's mathematics effort.

Poynter, L. (1999). Teacher advice on connecting school and home. *ENC Focus, 6*(1), 34–36. URL: <http://www.enc.org/topics/innovate/implementing/documents/0,1946,FOC-000722-index,00.shtm>.

With a focus on the role parents play in supporting curricular reform in mathematics, this article relays the steps one veteran elementary teacher took to ensure that parents were involved in and understood their children's mathematics learning. The teacher, experienced in using the *Investigations in Number, Data, and Space* curriculum, made early contact with parents at an open house in her classroom. She worked to help parents see the value of their children's mathematical projects and made herself available to help parents think about working on mathematics with their child. The article contains a variety of practical tips for working with and communicating to parents about mathematics curriculum and instruction.

Seo, K. & Bruk, S. J. (2003). Promoting young children's mathematical learning through a new twist on homework. *Teaching Children Mathematics, 10*(1), 26–31. URL: <http://static.highbeam.com/t/teachingchildrenmathematics/september012003/promotingyoungchildrensmathematicallearningthrough/>.

Seo and Bruk describe a homework project created at an early childhood public school in Milwaukee to support young children's (ages 3–5) mathematics learning both at school and at home. In addition to wanting to expand children's mathematical experiences and increase their familiarity and comfort with games and hands-on activities, the principal and teachers wanted to address parent concerns about improving basic skills. However, they wanted to do this without using traditional worksheets that did not align with their curriculum or their values. The teachers had worked with parents to help them understand the curricula being used at the school, *Investigations* for K–5, (adopted by Milwaukee public schools) and a teacher developed curriculum based on the NCTM *Standards* and other guidelines for young children for K–3/4, and they wanted homework activities to match this philosophy. Teachers identified key mathematics concepts from each curriculum and developed four activities that would involve parental support and engage the children. Activity sheets were created to help parents understand the tasks and provide guidance to their children. The activity sheets also provided space for the children to record their thoughts and reflect on their experiences. Inexpensive activity kits gave students access to hands-on materials (e.g., crayons, paper clips, etc.) at home. Introduction to the math homework project occurred during parent orientation and parent conferences, and communication was encouraged throughout the year. Return rates were consistently between 50% and 80% and both parents and teachers had enthusiastic reactions and reported changes in student understanding. Teachers also noted that their relationships with parents had been transformed, and that parents had developed strategies for assisting with and assessing their children's mathematical learning.

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