Chapter 1

What Is a Standards-Based Curriculum?

Talk about "the *Standards*" is everywhere. You hear about them when you go to conferences, read professional publications, receive guidelines from your state department of education, talk with colleagues in the teachers' lounge, and discuss curriculum options with publishers. While people often talk about the *Standards*, it is not always clear what they mean. Different people focus on different aspects of the *Standards* and may interpret them differently as well.

The NCTM has presented a view of mathematics learning, teaching, and assessment that shifts the focus away from memorization and rote application of procedures toward standards for performance that are based on conceptual understanding and reasoning. There are several forces promoting this shift, including recent advances in theories of learning and the dawning of the information age. The NCTM developed a set of national standards for curriculum, instruction, and assessment in order to answer several questions:

- What do we want students to know about mathematics, and what should they be able to do?
- How do we determine when students know the mathematics we want them to know?
- What mathematics do we want teachers to know and be able to do?

The responses to these questions are contained in the three *Standards* documents referred to in the Preface, representing the consensus of leading mathematics educators nationwide. These documents are based on a set of beliefs about mathematics as a body of knowledge and about the learning processes that lead to mathematical understanding. Briefly, these beliefs include the following:

- Mathematical literacy is essential to becoming an informed and competent citizen.
- All students can (and should) become mathematically literate, not just those students who have traditionally performed well in mathematics classes.
- Literacy involves understanding mathematical principles (such as change, function, and quantitative relationships), developing mathematical ways of thinking, and developing fluency with number, geometry, and data.
- Students develop this literacy by actively doing mathematics—using their skills and knowledge to solve problems and investigate mathematical ideas.

Many leading mathematics educators interpret the relatively poor showing of U.S. students in the recent Third International Mathematics and Science Study

(TIMSS) as evidence of the need for a change toward standards-based mathematics instruction. They argue that student performance on the TIMSS evaluations indicates a weakness in students' conceptual understanding and mathematical reasoning abilities, and that one source of this weakness is the curriculum, which has been described as "a mile wide and an inch deep."

In an effort to create classroom materials teachers could use to promote deeper and more substantial mathematical understanding in their students, developers of standards-based curricula grounded their work in the core beliefs outlined above. Building from these principles, different curriculum developers have emphasized somewhat different aspects of the *Standards* and have taken somewhat different approaches in their materials. For example, some programs place a heavy emphasis on student discourse, some use applications to motivate the mathematics in the curriculum, some embed skill development within the context of real-world problem solving, and some include more explicit practice. The result is a collection of mathematics programs that bear a "family resemblance" to one another because they are based on a common set of core beliefs, but which also represent distinctive interpretations of those beliefs.

A large number of currently available curricula describe themselves as "standards-based," and even a cursory examination of the presentation of materials will confirm that virtually every text looks different than those of 30, or 20, or even 10 years ago. All but the most traditional of today's textbooks offer sections on problem solving, include applied problems that involve practical uses for the mathematics students are learning, cover mathematical topics that were not part of textbooks 15 years ago, and presume that their users will come from a variety of backgrounds, bringing a range of learning styles to the mathematics classroom. Yet there is a significant difference between texts that have retrofitted their traditional "demonstration and practice" approaches in order to better align themselves with the NCTM *Standards*, and curricula that were designed from the outset to embody the mathematical approaches and pedagogical principles advanced by the *Standards*.

This section reviews the key aspects of the NCTM *Standards* that relate to curricula, in order to help you recognize the differences between curricula in which the goals and values of the *Standards* are integral to the fundamental design and current curricula that have imported some of the activities and instructional techniques associated with the reform movement.

The *Standards* identify four essential aspects of mathematics education and articulate goals for each of them with respect to curricula. The four aspects are:

- mathematical content
- mathematical processes
- attitudes toward mathematics
- views of teaching and learning

Following is a brief explanation of each, along with some implications for the teaching of mathematics.

Mathematical Content

First and foremost, the *Standards* are about the ideas and skills that children should acquire during their K–12 mathematics education. The *Standards* stress the importance of helping students develop deep conceptual understanding, fluency with skill-based manipulations, and the ability to reason and communicate about mathematical ideas. They emphasize the understanding of underlying mathematical concepts and the connections among them—the "big ideas" of mathematics that children learn over a long period of time, across topics and units. Curricula can support student understanding by drawing connections among mathematical ideas and between mathematics and everyday experiences, rather than by presenting mathematics as a set of discrete, unrelated topics that students learn, forget after the test, and then (perhaps) relearn the next year.

This perspective on the importance of conceptual understanding and the connectedness of mathematical ideas led the authors of the *Standards* to recommend less rote learning. For example, they advocate spending less time teaching such procedures as long division or the factoring of polynomials and place a greater emphasis on spatial reasoning, understanding probability, and reasoning about data. These changes are motivated by beliefs about the kinds of mathematical skills and understanding that will be needed for life and work in the twenty-first century.

Big Ideas

"Big ideas" in mathematics are those ideas and principles that govern the structure and functioning of the mathematical system. When students understand these "big ideas," they have the conceptual tools to approach and solve many different kinds of problems. One of the fundamental ideas of algebra, for example, is that you can operate on an unknown number as if it were known. The ability to think about mathematical relationships in terms of the general case allows students to summarize observations, make predictions, and develop such proofs as the following:

> *Prove that the sum of any two consecutive numbers is odd.*
>
> If the first of the two consecutive numbers is n, then the second is n + 1.
>
> The sum of the two numbers is n + n + 1, or 2n + 1.
>
> 2n is even, regardless of the value of n.
>
> Adding 1 to any even number yields an odd number, so the sum of any two consecutive numbers must be odd.

Some have interpreted the *Standards*' emphasis on conceptual understanding to mean that they do not recommend traditional skill mastery. This is not the case. The *Standards* do *not* contend that computation is unimportant or that students can get by without knowing basic number facts and operations. They do, however, recommend diminishing the amount of class time spent on strictly rote skills development (the "drill and kill" approach) in order to make more room for conceptually-based learning. Some of the newer programs embed mastery of skills in games or activities that also target other kinds of thinking (for example, developing strategic thinking or number sense). Because skill mastery is somewhat hidden in these contexts, someone who is unaccustomed to recognizing the skill component of standards-based curricula may have the impression that skills and facts are not being taught.

Below is a list of the general content areas recommended in the original *Standards* document. For a more detailed version, see the *Curriculum and Evaluation Standards for School Mathematics*. Note also that the upcoming revision, *Principles and Standards for School Mathematics*, will include changes in recommendations about specific mathematical strands. What will not change is the emphasis on developing solid understanding of important mathematical concepts, rather than a passing familiarity with many mathematical topics.

Grades K–4 Content Areas	Grades 5–8 Content Areas	Grades 9–12 Content Areas
Number Operations/ Computation	Number Operations/ Computation	Algebra
Geometry/Measurement	Geometry/Measurement	Geometry
Probability/Statistics	Probability	Trigonometry
Patterns/Relationships	Statistics	Functions
	Algebra	Statistics
		Probability
		Discrete Mathematics

One criticism of the *Standards* is that the recommendations simply add new areas to an already long list of topics to teach. Developers of new standards-based programs have sought to focus on the most important mathematical ideas, choosing topics that support their development. Nonetheless, your district will need to have a clear sense of the mathematics you want your students to learn so that you can examine potential curricula in this light. Reviewing your district's current curriculum scope and sequence, the kinds of assessments your students must take, and relevant standards and frameworks (which may include national, state, and/or local documents) can help you decide on the content that is most important for your students to master.

Mathematical Processes

Students gain mathematical competence by learning to think about mathematics and communicate their ideas to others. In addition to mastering skills and concepts (the major focus of traditional mathematics education), students should be engaging in a variety of mathematical processes. The *Standards* outline four processes that promote mathematical thinking across the grade levels and which should be a fundamental part of mathematics education:

1. **Problem solving.** The *Standards* recommend that students learn to develop and apply strategies for investigating mathematical content and solving problems. It also is important that problem solving be contextualized, that is, that the problems be motivated by situations and applications that give them meaning.
2. **Communication.** The *Standards* emphasize the importance of written and verbal communication. Expecting students to convey their mathematical ideas to others encourages students to reason clearly, articulate their thinking and justify it to others.
3. **Mathematical reasoning.** There is great value placed on learning to think logically and critically about mathematics from the very earliest grades. The emphasis on reasoning about mathematical situations begins with students recognizing and effectively using different problem-solving strategies in the early elementary grades, and leads to students employing both inductive and deductive reasoning by high school.
4. **Connections.** The *Standards* also emphasize the importance of learning to make connections, both within mathematics and between mathematics and the world. Students should make connections between different mathematical strands, different representations or models for the same mathematical relationships, particular examples and general mathematical principles, personal experiences and mathematical situations, and practical problems and mathematical solutions.

In order to give you a flavor of the kind of thinking that the *Standards* seek to encourage, we have included an excerpt of a classroom vignette written by a fourth grade teacher. In this excerpt the teacher describes a conversation she had with one of her students, a young girl named April, as April posed and solved a division problem. As you read this vignette, look for the ways that April works to understand division—thinking about groups of objects, finding upper and lower bounds for her answer, breaking down the original problem into more manageable units, and using diagrams to visualize a solution. Be aware as well of how the teacher works to understand April's thinking and to help her continue developing her solution.

> [April] started off by saying that "you have to know multiplication [to do division]" . . . I asked her to work on another problem so she could talk a little more about how her strategy developed . . . She decided to use jelly beans (being Easter and all!) again and make the problem a little harder.

Divide 143 jelly beans among 8 kids.

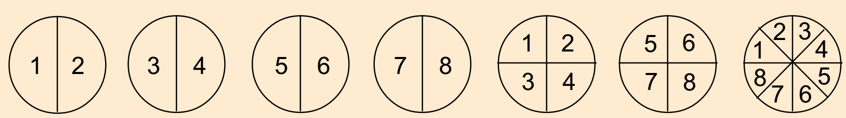
I asked April to tell me first what she did when she was dividing. She said that division was putting things into groups. “You have to multiply the things to get the groups—like take 8 times any number and see what the answer is.” She began her strategy by saying that she knew each kid would get more than 10 jelly beans because that would be 80, and that if each kid got 20 jelly beans, that would be 160, which was too much. So she knew two things: that 10 was too small and 20 was too big. I asked if she thought the correct number would be closer to 10 or closer to 20. She said she thought it would be around 14, and proceeded to multiply 14 x 8, 112. She then said that you had to add 31 more. She then decided to try 22 x 8 and got 176. She said she knew she had to try a number that would bring her close to the 160 mark, but lower.

April now decided to “bag” this idea and came up with another strategy—to see how many eights were in 100, and then how many eights were in 43—to see if that worked. Her process:

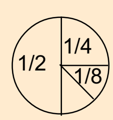
> 10 x 8 = 80, 11 x 8 = 88, 12 x 8 = 96 (with 4 left)
> 5 x 8 = 40 (with 3 left)

The next part she struggled with. She was losing her train of thought and was confused by all the numbers she had just generated. I tried to help her sort out what she had done by showing her that she had found 12 groups of 8 in 100 and 5 groups of 8 in 40, and that in both cases she had numbers left over. I asked her to think about how many groups of 8 she had. She had 17 groups of 8 (which totaled 136) with 7 jelly beans left over . . .

Then she came up with a way to divide up the 7 extra jelly beans. She took 4 of them and divided each in half, so each of the 8 kids got 1/2. Then she had 3 left over, so she took 2 of those and divided them into fourths, so each kid got an additional 1/4. Then she divided the last jelly bean into eighths, so each kid got another 1/8.



Now the question was how much was 1/2 + 1/4 + 1/8? This is how she solved that problem.



April was delighted when she saw that 1/8 was left over; she immediately knew that the total was 7/8. In the end, each kid got 17 7/8 jelly beans.[2]

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[2] Excerpted from “Janie, 4th grade, April-May: Sharing Jelly Beans,” in Schifter, D., Bastable, V., & Russell, S.J. (1999). *Developing Mathematical Ideas Casebook: Number and Operations: Building a System of Tens*. White Plains, NY: Cuisenaire • Dale Seymour.

By emphasizing the development of mathematical processes, the *Standards* stress the importance of developing mathematical thinking by engaging in mathematical work. A number of years ago, chess Life Master John Collins talked about a "chessical" way of thinking—the ability to read a board, have a certain appreciation for the flow of play, and command the interplay of strategy and tactic. Chess players learn to think chessically by immersing themselves in the play and study of chess. Similarly, students develop mathematical ways of thinking by immersing themselves in the exploration and study of mathematics.

When these content and process *Standards* are taken together, they point to a very different kind of mathematics curriculum—one that is organized to address the development of conceptual, as well as procedural, understanding; follows the thread of ideas through mathematical strands; poses different kinds of problems to students; and promotes a different kind of problem solving. As you review standards-based programs, you are likely to find that they have the following features:

- There is a more integrated approach to topics, with several areas of mathematics appearing at each grade level and developing in connection to each other rather than in isolation. Students work on ideas about number, function, geometry, and data from kindergarten on. This is particularly noticeable at the high school level, where the traditional Algebra I–Geometry–Algebra II–Trigonometry/PreCalculus series has been modified to create a more interconnected and integrated sequence of mathematical topics throughout grades 9–12.
- Topics reappear at different grade levels in increasingly sophisticated forms. For example, early elementary-level lessons on probability and statistics focus on ideas of chance, developing in later grades into the study of fairness and statistics; elementary grades' work with patterns evolves toward a more formal study of algebraic relationships in middle and high school.
- Mathematical knowledge is developed within practical and conceptual contexts. There is less work with "naked numbers"—decontextualized problems whose goal is symbol or number manipulation—and more work that connects problems to other mathematical ideas and the world.
- Many problems are complex, requiring a number of mathematical ideas and skills and taking more time and thought to solve than the problems of the past.
- There is an emphasis on using different kinds of representations, such as charts, tables, graphs, diagrams, and formal notation, for exploring, describing, and testing problem situations.
- Teachers and students take on different roles in the classroom as more of the learning occurs through exploration and discussion of mathematical ideas. Students make conjectures and investigate them, explain their reasoning, and work individually and in groups to solve problems.

- Teachers lecture less and listen more for students' ideas, ask questions designed to clarify and extend students' thinking, and pose new questions or activities to further their understanding.

Attitudes Toward Mathematics

"I have horrid memories of learning mathematics as a youngster. I can vividly recall crying my eyes out because I couldn't do long division."

"I've never liked math. A lot of it has to do with self-confidence. I never felt I really understood math. I just did what it took to get by."

"I hated math as a kid. I got math facts wrong. I didn't understand what was going on. Algebra was a mysterious language that was undecipherable."

It's not unusual for people to associate their experiences as mathematics learners with confusion, uncertainty, and discomfort and to think of themselves as deficient mathematics learners. In fact, many adults recall with relief the day they endured their last mathematics class. Recently we asked a group of people in their twenties, thirties, and forties to write mathematics autobiographies; two-thirds of them wrote with surprising intensity about their negative experiences and feelings!

The *Standards* call for education that will create a positive shift in attitudes about mathematics for all students. Instead of developing the belief that mathematics is mysterious, if not downright unfathomable, students should come to think of mathematics as an interesting, sensible, and useful body of knowledge.

Equity in mathematical mastery is another major goal of standards-based curricula. Historically, mathematics has often been considered a subject that is understood by a select, especially talented few. The *Standards* take a contrary view, emphasizing the critical importance of making mathematics accessible to *all* students—those who have traditionally excelled and those who have struggled or simply tuned out. The *Standards* call for engaging all students in educational experiences that will enable them to recognize and value the power of their own mathematical thinking.

The developers of standards-based curricula have sought to address issues of equity in a number of ways. At a superficial level, they have depicted people from different racial and ethnic groups and people with disabilities within the materials themselves. More substantively, they have worked to create lessons that have multiple entry points, allowing students with different levels of mathematical sophistication and different learning styles to find ways to engage with the mathematical ideas (for an example, see the "Crossing the River" problem in the box below). Developers have also sought to motivate mathematical work by presenting problem contexts that students are likely to find interesting and compelling.

Crossing the River

There are eight adults and two children having a picnic on the banks of the Lazy Horse River. They finish their picnic and want to go exploring on the other side of the river. They have one small boat and everyone can row it. It can hold one adult, or two children at a time. It can also hold just one child. The river isn't very wide or *very deep and the children are good rowers, so it's okay for them to row back and forth on their own.*

A. How many one-way trips does it take for the entire group to cross the river?

B. How many trips would it take for 2 children and 100 adults?

C. Describe how you would figure it out for 2 children and any number of adults.

Below are solutions from three middle-grade students. These solutions illustrate different approaches to the problem, all of which yield the correct answer. Students generally devised a strategy to solve the initial problem (part A) and then generalized this strategy to construct their own rule to solve the extensions of the problem (parts B and C). In the first solution, the student describes a rule for counting the total number of trips. The second solution uses a traditional algebraic representation, while the third student describes the solution with a diagram and written explanation.

Solution 1:

One student answered part B as follows:

"It takes four trips to get 1 adult across the river. And one additional trip for the children to get across the last time. For 100 adults to get across the river (including 2 kids) it would take 401 total one-way trips."

Solution 2:

In solving part A, the student answers, "Begin with 3 trips to get 1 adult over. Each additional [adult] is four trips." Then she constructs her rule for part C by stating, "Subtract 1 from the total number of adults. Multiply that answer by 4. Add three to the total of that. Add two to get the kids across and there's your answer!

(A-1) * 4 + 3 + 2 =___"

Solution 3:

This student answers part B by describing an algorithm and illustrating it with a diagram:

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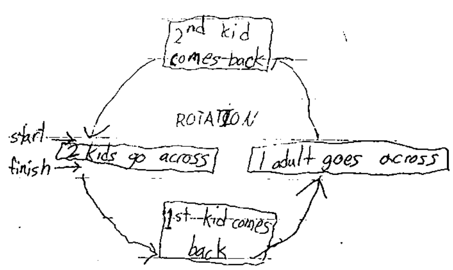

The rotation begins at the start arrow. It occurs once for every adult. When all of the adults are across, the 2 kids go across for the final time.

For 100 adults and 2 children it would take 401 trips

Views of Learning and Teaching

The *Standards* take a view of learning and teaching that adds certain dimensions to traditional mathematics instruction. The emphasis on engaging students in *doing* mathematics—making connections, problem solving, reasoning, and communicating—is intended to help students understand the *why* as well as the *how* of the mathematics they study.

This emphasis on students' active learning derives from the fact that learners *construct* their own understanding through their experiences with mathematical problems and discussions with teachers, parents, and peers. They are naturally driven to use their current knowledge and understanding to make sense of situations and work toward accurate and efficient problem solving.

In order to support students' construction of deep and flexible understanding of mathematical content, the *Standards* recommend that students of all ages do the following:

- interact with a range of materials for representing problem situations, such as manipulatives, calculators, computers, diagrams, tables, and charts
- work collaboratively as well as individually
- focus on making sense of the mathematics they are studying as well as learning to achieve accurate and efficient solutions to problems

These recommendations mean that instruction in a standards-based classroom has a flow and focus that may be unfamiliar to teachers. Classrooms are often busy—students talk with the teacher and with each other (sometimes even debating heatedly), leave their seats to get materials or to consult with others, cover their desks with the materials they need to represent problem situations, explain and defend their ideas, and sometimes work on problems over an extended period of time (days, or even weeks). Children may approach the same problem in very different ways, sharing and comparing their various strategies. There is more action and noise and, to an unaccustomed eye, such a classroom may look bustling—perhaps even unfocused. After some practice, however, it's possible to recognize busy, but nonetheless purposeful, learning.

Teachers, too, have different roles in standards-based classrooms. They listen closely to their students' ideas, analyzing them to learn where student understanding is firm and where it is still developing. They use this careful listening, along with their knowledge of how children's mathematical thinking develops, to plan lessons to move their students' understanding forward. Teachers are more aware of the need to ask questions that will lead in mathematically productive directions and to give their students the space to think through ideas on their own. The curriculum materials themselves can help prepare teachers for these roles and responsibilities, but they cannot script the conversations or prescribe the implementation strategies that will guarantee student learning.

Teachers in standards-based classrooms must become more alert to student thinking, more cognizant of the “big ideas” that underlie the mathematics they teach, and more focused on helping students understand and apply these ideas. For many teachers, developing these skills and strategies will require time and support through professional development.

Summary

Standards-based curricula are built around the principles and perspectives articulated in the NCTM *Standards* documents, which include the importance of the following:

- mathematical content
- mathematical processes
    - ~ problem solving
    - ~ communication
    - ~ mathematical reasoning
    - ~ connections
- attitudes toward mathematics
- views of teaching and learning

Curricula that are developed around these principles differ from conventional programs in both their approach to the mathematical content and their pedagogical stance. Learning to evaluate standards-based curricula, therefore, involves learning to look at materials differently.