The K-12 Mathematics Curriculum Center

Issues in Mathematics Education

Thinking (And Talking) About Technology in Math Classrooms

E. Paul Goldenberg, Education Development Center, Inc.

One of the strongest forces in the contemporary growth and evolution of mathematics and math teaching is the power of new technologies. In math, computers have fostered entirely new fields. In education, they've raised the importance of certain ideas, made some problems and topics more accessible, and provided new ways to represent and handle mathematical information, affording choices about content and pedagogy that we've never had before.

But such choice imposes the burden of judgment. Not everything that *can* be done *should* be done. Student learning is affected by a complex system: teachers, educational theories and beliefs, parents, curricula, students' interests and aspirations, resources, cultural expectations, technology, and more. There is much to say about all of these considerations, but the impact of each cannot be fully understood except in relation to the others. This is especially true of technology, which partially explains why there is no single, universally accepted view of the best use of calculators and computers in classrooms.

Moreover, the right questions about technology are not broad ones about which hardware or software to use, but about how each works in a certain curriculum, right down to its effect on how individual *problems* are posed to the student. Each of these is its own unique case to judge as effective and appropriate or not. The need to make decisions at that level of detail is unsurprising if we think of computers and calculators the way we think of pencils. It is the *problems* that are posed, not the technology with which they are attacked, that make all the difference. With computers, as with pencils, some problems are great and some are a waste of time.

With technology, what changes is the pool of problems to choose among and the ways they can be presented. Some problems are too hard to be posed in a pencils-only classroom. Some lessons require students to experiment with certain mathematical objects and see how they respond. Some require visual representations—graphs, diagrams, geometric figures, moving images—that *respond* to students' questions, answers, or commands.

In the early grades, physical manipulatives often provide these visual and experimental supports for children. They serve as temporary physical stand-ins for mathematical ideas, objects that the children can see with their eyes and manipulate with their hands while they learn to see and manipulate the mathematical ideas with their *mind's* eyes and hands. In the higher grades, many mathematical ideas just don't have such physical models. Computers can provide interactive "virtual manipulatives" where physical devices do not exist. As always, the value of a tool depends on how it is used. If physical or electronic manipulatives are well designed and well used, they can increase the variety of problems that students can think about and solve.

But what *is* good use? Research fills in parts of the picture, but can't give all the answers. For one thing, goals differ from district to district and even from classroom to classroom. Clarity about goals and good teacherly judgment are both necessary components of decisions about the use of any teaching method. This paper aims to help you decide how you stand on whether, when, and how to use computers or calculators, and how to maximize the gains and minimize the risks of their use. It also aims to offer you strategies for talking with parents about the choices you make.

Some Ways to Decide What is Good Use

The single most important thing that research shows is that what really matters is not the use of technology, but *how* it is used. (This is likely to seem obvious to any teacher but is, alas, not the way the debate tends to be carried out in public arenas.) Despite the research, no document can prescribe good or bad practice, because so much remains a matter of personal and community judgment. Even so, there appears to be enough agreement, even between those who love technology and those who hate it, to come up with some reasonable principles that may help you make your own decisions. Below are six such principles for thinking about technology use in math classrooms. They all point to the need to examine the *purpose* of the lesson—that is, the nature of students' current needs—and how the technology fits with that purpose.

While curriculum is the real determinant of what mathematical ideas students gain, from the perspective of providing students with mathematically rich, responsive environments for encountering, representing, experimenting with, and reasoning about mathematical ideas, computers generally offer greater versatility than calculators. Sheer screen space alone makes possible a broader range of mathematical ideas, with more ways to represent and manipulate them. That versatility also accommodates a greater variety of learning and teaching styles, offering such educational settings as puzzles, microworlds (environments designed specifically for education, but tool-like in their structure), tutoring systems, mathematical programming environments, visualizations in mathematical domains from statistics to calculus, geometric construction tools, and more.

Despite the variety, mathematical fidelity and power, and educational appropriateness of many computer-based tools, economic and other reasons have moved calculators into schools and curricula on a scale that is, so far, unmatched by computers. Controversy about the uses of technology in mathematics classrooms has therefore also tended to focus primarily on calculators. The emphasis in this paper will be proportionate not to the opportunities, which seem vastly greater with the computer, but to the current classroom realities, in which calculators, at least for the moment, still dominate.

"Higher-Order" or "Lower-Order" Skills? Choosing a Genre of Technology

A very readable 1998 report by Educational Testing Service (ETS) outlines the debate on technology's effectiveness and concludes that the use of computers to teach higher-order thinking skills such as puzzling, reasoning, and problem solving was "positively related to academic achievement in mathematics and the social environment of the school."[1] Furthermore, "use of computers to teach lower-order thinking skills [e.g., learning facts] was negatively related" to the same two variables. It also shows that the nature, not the frequency, of school use was the critical factor.

Most people agree that learning facts is necessary—note that the ETS study ascribes harm not to the learning of facts, but to the use of technology to serve that goal. And most teachers agree that developing rapid, easy access to useful facts (sometimes called fluency) requires practice. Computer games are sometimes used to provide drill without the mindless monotony students sometimes experience on worksheets. But whatever motivational or other value computers might seem to provide for achieving that practice, they are apparently *not* the way to go. The research did not say why computers in support of lower-order thinking was harmful, so we cannot draw a definitive general principle other than "don't use computers as flashcards."

What does the distinction between using tools for higher-order and lower-order thinking *really* mean? The answer involves several factors. Some of what is higher-order thinking at one stage becomes lower-order at a later stage, so students' developmental level and mathematical background are part of the picture. The nature of the problem matters, as well: if one pays for a 75¢ item with a dollar bill, there is just more to think about in deciding how many ways the change can be given than in deciding what change to give.

The technology matters, too. Students are in a different position when the software poses problems they must solve—tutoring, drill software and many games are like that—than when the software waits for the *students* to formulate (and solve) the problem. Word-processors sit blankly, waiting for the writer to have ideas, and to express, develop, and edit them. The real work is being done by the writer. In a similar way, there are mathematical environments—they might be dubbed collectively "mathematical idea-processors"—that *also* sit and wait for the student-mathematician to have ideas, and to express, develop, and edit them. These include programming languages, dynamic geometry software like *Geometer's Sketchpad* or *Cabri*, spreadsheets, symbolic algebra calculators, and the like.[2] As in a good writing class, the teacher and the curriculum play a central role: students don't just "have" ideas, but need some good problems to work on, some skills and ways of thinking, a developing arsenal of strategies and techniques, facts, and guidance. But with "idea-processors" like word-processors, spreadsheets, programming languages, and geometry construction software, and with certain microworlds especially tailored for educational purposes—the statistics program *Fathom*, for example—the quality of the problems can be under the control of the teacher and the curriculum, not the software designer.[3] Moreover, the problem-solving approach can be under the control of the student, with guidance from the teacher.

Is idea-processing the only recommended use of technology? No. While the ETS study warns against a focus on practicing lower-order thinking skills, some brilliant puzzles and games are clearly aimed at higher-order thinking, even though they do pose the problems and leave kids merely to solve them. And for some students under some limited circumstances, there is even a role for software that gives practice with elementary arithmetic skills.

The Genre Principle: *Good decision making requires us to be aware of these different roles for technology, to think clearly about our own classroom goals, right down to the particular needs of particular students, and to choose technologies expressly to further those goals, rather than merely adding technology to the classroom in ways that may be attractive but tangential or even detrimental to the goals we set.*

Look at the Purpose of the Lesson

A well-designed lesson has a central idea and focuses students' attention on it, without distraction by extraneous ideas or procedural details. So, for example, when middle schoolers are asked to compare the ratio of two measured quantities with the ratio of two others, the likely purpose of the lesson is to develop their ideas about ratio and proportion. With *that* purpose, the actual mechanics of performing the many calculations involved could constitute a distraction. Many teachers as well as education theorists readily agree that in situations like this, using a calculator to avoid long decimal divisions makes sense. The lesson is not *about* division, so the calculator doesn't short-circuit it; the lesson is about ratio, and electronic aids in calculation help focus it by reducing the effort spent on computation.

Of course, what bogs students down in computational details depends not just on the size of the computation, but also on what the students find easy to do, so judgment is still required. For six-year-olds, figuring out how many inches are in 20 feet could be an excellent problem to ponder without a calculator. Alternatively, it could be one difficult and distracting step in a process in which some other step is the current focus of their learning. In this case, calculator use might reasonably be warranted. By the time students are ten they may be able enough to multiply 12 times 20 in their heads so that the calculation process is not a real distraction from the point of the lesson, even if it is *not* the point of the lesson.

What irks people is the habit that some students develop of pulling out their calculators to divide by 10 or 100, or to perform other such calculations that virtually everybody should be able to do in their heads. With very few quite extraordinary exceptions, people who actually need to use a calculator for tasks like these have been handicapped by inappropriate education, not by any fixed limit to their "native" ability.

The Purpose Principle: *Allow calculator use when computational labor can get in the way of the purpose of the lesson. When learning how to perform the computation is the purpose of the lesson, calculators may be a bad idea.*

Look at the Nature of the Thinking Being Asked of Students

In the example above, allowing technology to perform a computation freed students *not* to think about the computation so that they could focus attention on some other aspect of the problem. But sometimes, even if some process such as computation is not the point of a lesson, analyzing the process can be a natural route to understanding (rather than just memorizing) the main point. Here is an example. Many curricula ask students to know that rational numbers have repeating or terminating decimal expansions. Take 30/7 as a case in point. Dividing 30 by 7 on my calculator, I see 4.285714286. My computer's software calculator gives me a few more places—4.28571428571429. Even though both round the last digit, the repetition is quite noticeable. In a lesson whose goal is to *find* the pattern, such computational tools are much more convenient than hand-calculations, but they give no direct insight into what underlies the repetition. *Why* do rational numbers have repeating or terminating decimal expansions? Opinions may differ about whether this fact about rational numbers is important to learn, but if it is considered an important idea, then the traditional division algorithm is a useful way to move beyond rote and help students understand why it is so. Here is a close-up look at the minute details of how the decimal expansion of 30/7 is worked out using the division algorithm.

```
     4.28571428
 7 )30.00000000
   -28
     2 0
    -1 4
      60
     -56
       40
      -35
        50
       -49
         10
         -7
          30
         -28
           20
          -14
            60
             .
              .
               .
```

Using the familiar goes-into and bring-down language, 7 goes into 30 four times, leaving a remainder of 2. Bring down a zero. Seven goes into 20 two times and leaves a remainder of 6. After a few more steps we see . . .

another “7 goes into 30.”

Why? Because either we reach a point where there’s no remainder (and the decimal terminates) or we get a remainder. As only six possible remainders exist, they must somehow repeat. Whenever they do, a brought-down zero starts a process we’ve already seen before. The remainders must cycle, and so the quotient digits repeat.

Although the *example* is only about 7 and 30, the *reasoning* is completely general—it does not depend at all on the numbers that were selected. At each step in the division, either there is *no* remainder and the decimal terminates, or there *is* a remainder. When dividing by n, remainders must be smaller than n. If there is no remainder (if the “remainder” is zero), the division terminates. So only n–1 non-zero remainders are possible. When the division does continue, only zeros are “brought down,” so the n will be divided into at most n–1 different numbers (all multiples of 10). Therefore, in at most n–1 steps, the process must either end or repeat a division that was performed earlier with exactly the same steps following it—a repeating cycle.

Because computational support can ease the job of obtaining initial results and may reduce annoying errors that would obscure the pattern, it might be of use at the stage at which students are just generating a conjecture. But understanding the *reason* for the pattern may require them to perform the division algorithm at least once in order to see the intermediate results. For this goal, the calculator is no longer a help but a hindrance.

The Answer vs. Analysis Principle: *The Purpose Principle said that calculator use was okay when the lesson’s purpose was better served by getting quickly to the answer rather than laboring over or being distracted by the computation. At other times, even when the process of calculation is not the point of the lesson, stepping through that process and seeing the intermediate details explains the results that it produces. At such times, a technology that obscures the details and skips directly to the answer is no help.*

Look at the Role of the Technology in the Lesson or Activity

An even more general truth is lurking here. *Anything* that influences how a problem is solved—for example, using paper and pencil to work a computation instead of performing it mentally, or using ruler and compass to work a geometry problem instead of using either dynamic software or a freehand sketch—highlights certain aspects of the problem and suppresses others. Only when one is clear about the *learning goal* of the problem can one make a clear decision about what technology (mental, paper and pencil, electronic, etc.) to use. For example, while using the standard paper and pencil algorithm for division seemed an excellent way to understand why decimal expansions of rational numbers repeat or terminate, using the standard paper and pencil algorithm for multiplication (or using the calculator, for that matter) probably does *not* give much real insight into why multiplying by 10 moves the decimal point or adds a zero at the end. Using the calculator might establish the pattern well enough to allow students to feel comfortable with it and be fluent at using it as a “fact.” But if insight into how it works is desired, a different representation may work—perhaps a physical, visual, or software representation of the reasoning,

$$0.3 \times 10 = 3 \times 0.1 = 3 \times 1/10 \times 10 = 3 \times 1 = 3$$

or perhaps that reasoning presented just as is, purely symbolically. Or one might appeal to a demand for consistency in notation: if five tens is written 50, and nine tens is written 90, then thirty-six tens *should* be written 360, and 4.3 tens *should* be between four tens and five tens (with details to be worked out as in the case of 0.3 x 10). Again, a visual representation might help, and that representation might usefully be electronic and interactive, but a tool to *get* the answer is not what is needed.

The same is true in algebra. Students might investigate and then make a conjecture about a pattern of algebraic computations, using a calculator or computer to ease the labor in performing the symbolic manipulations. The technology might show them, for example, that

Further experiments might suggest a pattern, but how do they learn *why* this pattern holds? A calculator just gives the answer; insight and understanding lie in the intermediate steps that one takes when multiplying by hand. One misses them entirely when looking only at

$$\frac{(x^5-1)}{(x-1)} = x^4 + x^3 + x^2 + x + 1$$

answers produced by machine.

Here's one way of writing down what the calculation looks like by hand.

$$(x-1)(x^4+x^3+x^2+x+1) = x(x^4+x^3+x^2+x+1)$$
$$+ (-1)(x^4+x^3+x^2+x+1)$$

This breaks the calculation into two parts, multiplying by x and multiplying by -1. Although keeping track of the computation might require a by-hand component, the actual work is all *mental*, and the mental demands are absolutely basic to even the most minimal understanding of algebra. Students must know (and be comfortably fluent with) the effect of multiplying by -1 and the effect of multiplying by x. The next stages are straightforward. Multiplying by x gives

$$x^5 + x^4 + x^3 + x^2 + x$$

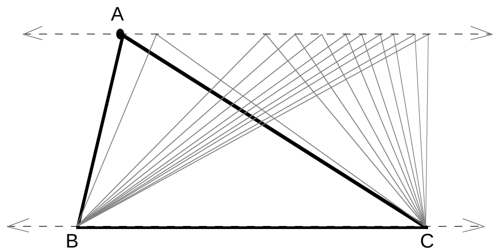
and multiplying by -1 gives

$$(-x^4 - x^3 - x^2 - x - 1).$$

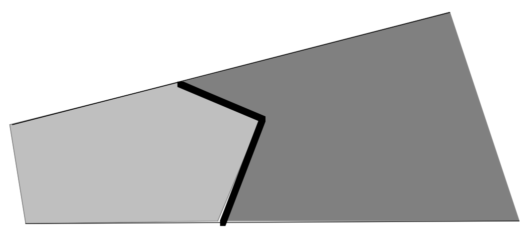
Adding these two expressions eliminates most of the terms, leaving only $x^5 - 1$.

Even though this is only one calculation—and we're told all the time not to base a proof on one example—it is immediately apparent that the way this example works is general enough to show us how any such calculation works, and to prove the conjectured pattern. *Not* using the machine helped.

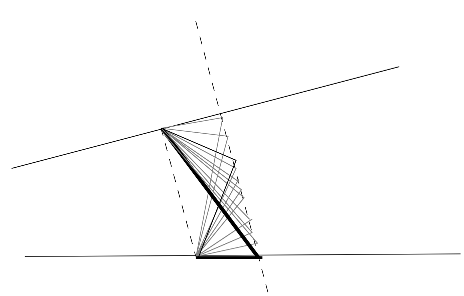
By contrast, *The Teaching Gap* and the related TIMSS (Third International Mathematics and Science Study) video give an excellent example of where a dynamic image, made practical by technology, supports student problem solving.[4] In summarizing the previous lesson about the area of triangles, a teacher shows his eighth-grade students an interactive image of a triangle with its base lying fixed on one of two parallel lines and its vertex lying on the other. As he moves the vertex (A) back and forth, he remarks about the fixed base and unvarying height, and reviews the main point that all such triangles must therefore have equal area.



He then moves to the new problem of the day, presenting it for the class to work on independently with no instruction about approach. "The boundary between two farmers' land (dark line in the illustration below) is bent, and they'd both like to straighten it out, but each wants to keep the same amount of land. Solve their problem for them."



Various solutions emerge. Some students recall the image of a triangle bounded by two parallel lines and see how to use it in this problem. They see the boundary line as two sides of a triangle, draw a parallel to the third side through the vertex, and adjust the figure—sliding that vertex along that parallel all the way to one side—to solve the problem. In this class, the image was a teacher-



manipulated demonstration, but tools like *Geometer's Sketchpad* and *Cabri* are age- and content-appropriate "manipulatives" that let middle- and high-school students perform experiments of this kind on their own.

Geometry tools, spreadsheets, symbolic calculators, and graphing tools can all be used to solve problems, but their value in that role is ephemeral: over time, the technology changes and students will not have those particular tools to use any more. In another role—helping students develop new and powerful ways of looking at problems, helping them build mental models, acquire generalizable and flexible skills, and so on—these tools can foster learning that does not evaporate as new products are developed. Students who watch carefully as they drag and distort geometric objects onscreen begin to learn how to perform the same kinds of experiments in their minds.

Even for elementary school students working with simple arithmetic calculators, there are activities, such as the "broken calculator" activities, that help students focus on and analyze the structure and elements of arithmetic and gain skills along with understanding, rather than have the calculator replace their thinking. The broken calculator activities are essentially puzzles in which students must figure out how to perform certain tasks on a calculator that has one or more non-working keys. At different stages of students' arithmetic learning, problems like "How can we multiply 20 times 50 if the zero key isn't working?" or "How can we multiply 5 times 12 if the only working number keys are 5 and 7?" or "How can we get 6 on the calculator if the only working number key is 5?" can be challenging, fun, and instructive.

The Who Does The Thinking Principle: *The Answer vs. Analysis Principle asks whether the technology is being used to solve a problem or to help students think about a problem, analyze a process, generate a proof. More generally, we might ask—specific to a lesson or even to a particular problem—whether the role of the technology is to replace a capacity that the student might otherwise need to develop or to develop the student's capacity to think, independent of the technology. Some of each may be warranted, but good use of technology depends on making such decisions consciously. This focus on helping students develop ways of thinking about a problem—thinking that eventually becomes independent of the technology—is reminiscent of ETS's conclusion that using technology to teach higher-order thinking skills was positively related to mathematical achievement, while using it to drill lower-order skills was negatively related.*

Keeping Control of Content

Technology is often said to make certain content obsolete. We no longer teach how to interpolate from tables of trig functions, logs, and square roots. Maybe, argue some, we should also ditch the division algorithm. With calculators so common, cheap, fast, and accurate these days, the hands-down favorite "real life" method for performing a calculation like 463.75 ÷ 29.41 is to go for a calculator if any precision is needed. Mine just said 15.76844611. To check this answer for plausibility, all I need to do is estimate how many 30s there are in 460. Midway between 10 and 20 looks quite good. Done. Is more needed?

Not long ago, there was no choice; people needed the division algorithm to get accurate answers to division problems. Things have changed. Finding quotients with precision now requires nothing but a calculator: *that* role for the division algorithm *is* obsolete. Spending much time in school as a hedge against dead batteries seems a poor choice. But you've already seen that there can be other reasons to learn the algorithm. In fact, deciding if it should be taught isn't obvious; reasonable people can come to different conclusions.

In the days when we had to teach the division algorithm, we had no need to ask what secondary purposes might be served by the knowledge of that algorithm, and yet you've already seen one. Standard division is also a good first experience with infinite processes and an excellent example of systematic approximation and successive refinement. Just as technology is merely one element in a complexly interdependent system of factors including teaching, students, and so on, ideas like the division algorithm are complexly interrelated with other mathematical ideas. Pulling one idea out of the curriculum may have unexpected results. This is not a case of conservativism for its own sake. It *is* important to reconsider old practice, and we *do* sometimes need to scrap traditional content to make room for new ideas. But . . .

The Change Content Carefully Principle: *Decisions about what is or is not obsolete content must be made thoughtfully, attending not just to what technology can do, but to a careful analysis of what students need to be able to do—especially how they need to be able to reason.*

Learning to Be Power-Users of Technology

As we worry about the inadequate current state of students' mathematical learning, let us not forget that in the "good old days," students, however good they might have been at whole number computations, were *not* broadly good at math even up to the addition of fractions, let alone algebra. The majority of adults do not report having been good at math. In fact, one of the rationales for using calculators and computers is to change educational practice precisely because students were not masters of the old tools like algebra.

But empowerment requires control. If students were not masters of the old tools, it is no favor to give them new tools that they also do not master. Sometimes students do know enough algebra to solve a problem but still fail to use that knowledge because they lack the fluency or experience to use it effectively and confidently in problem solving. The same applies to electronic tools. Learning just enough about a spreadsheet to solve a particular class of problems and then moving on or learning a few construction tools on geometry software to illustrate a particular collection of geometrical facts, and then moving on leaves students limping users of the tools, not experts who could whip out the tool as needed to help reason about and solve a problem.

As schools plan their use of technology, they may want—just as with any curriculum—to take a developmental approach to tool use, an approach that chooses a limited number of tools, introduces them early, and uses them consistently, developing increased skill and sophistication over the years so that students ultimately become power-users of these tools in all of their mathematical learning. Of course, the math curriculum must still be about the math, not about the electronics. In a high-tech classroom as in a paper-and-pencil one, quality rests primarily in how much and how well students are learning to think mathematically, but effective use of the local technology (wood-pulp or electronic) matters, too. With paper, neatness and orderliness are important, as anyone can attest who has seen the errors students make just because they cannot decipher their own writing. With electronic tools, other skills are required. And with *any* technology, mental skills are needed. We must be thoughtful about giving students the skills they need to make fluent, effective use of the new tools we give them.

We must also provide time and opportunity for *teachers* to become fluent with the tools so that they can be flexible, use spur-of-the-moment good judgment in their classrooms, and not feel constrained by the tools or stilted by a lack of confidence in their ability to use them. This was another major but not surprising conclusion of the ETS report: teacher professional development in technology—and, we add, especially around tools that support higher-order thinking skills—mattered greatly.

The Fluent Tool Use Principle: *"Touching" several computer or calculator tools but not really mastering them may do more harm than good: it costs time and teaches little. Learning a few good tools well enough to use them knowledgeably, intelligently, mathematically, confidently, and appropriately in solving otherwise difficult problems makes a genuine contribution to a student's mathematical education.*

Keeping Learning at the Fore as Change Marches (Inexorably) On

It's hard to resist a bargain. With technology doubling in power and halving in cost every couple of years, our entire society tends to run along, seduced by the possibilities and driven by the momentum, barely keeping up.

The first big wave of computers purchased by schools wound up living in closets. That alone tells us that the initial push for technology did not for the most part come from the classroom. Schools are a big market, and the drive to keep them technologically "up to date" is in the economic interests of the makers of hardware, software, and internet service providers. Policy—even government policy—is often driven first by those interests. So, with only partial control over the brakes or accelerator for this rapid influx of technology into schools, the job left to us as educators is to steer attentively.

Technology affords important and exciting new capabilities that expand the options for what and how we teach. But there is another side to this coin. It is easy, in the excitement, to overlook constraints and limitations of technology that narrow one's vision of math and teaching. For example, it is cheaper to put graphing calculators in the hands of every student than to give students equivalent access to computers. Having bought into calculators, however, can have the effect of narrowing one's vision of math to what can be accomplished on small screens with graphs and their symbolic descriptions. A great deal *can* be accomplished that way, and reasonable people might choose that as what they *want* their math program to accomplish, but it is equally possible to become so involved in one form of graphing method that other applications and views of algebra are forgotten.

The same is true of teaching. While some teachers enjoy the high adventure of experimenting with novel tools, many others feel more creative when their attention is not divided between their craft—students, thinking, and subject matter—and what may seem like low-level technological details. For these teachers, the advantages of a new tool (even if they agree it has advantages) can be offset by the retooling they must undergo in order to use it fluently. Some of the newer curricula both establish a vision for the use of technology and provide resources that help teachers gain power and fluency with these new classroom tools.

What's to be done? Keep change under control. Provide instruction *and time* for teachers to become creative users of the technology they have. Keep a clear vision of what is *desired* of the technology, responsive to but not governed by changes in what is *possible* with the technology. Think foremost about what you want for your children—the goals of the particular classroom, and the needs of each particular student—and after deciding on your goals, then assess whether the tools are bringing you closer or distracting you away.

Endnotes

[1] This result concerns the effect of computers on eighth-grade achievement and is based on an analysis of data from the eighth grade 1996 National Assessment of Education Progress. Wenglinsky, H. (1998). *Does it Compute? The Relationship Between Educational Technology and Student Achievement in Mathematics*. Princeton, NJ: Educational Testing Service (ETS). Available on-line at: <http://www.ets.org/research/pic/technolog.html>.

[2] *Geometer's Sketchpad* software is available through Key Curriculum Press. *Cabri* software is available through Texas Instruments.

[3] *Fathom* software is available through Key Curriculum Press.

[4] Stigler, J. and Hiebert, J. (1999). *The Teaching Gap*. Glencoe: The Free Press.

About the Author

Dr. Goldenberg studied mathematics, psychology, and education at Brandeis and Harvard. In over thirty years in mathematics education, he has spent nearly two decades in classroom teaching—from his own second grade self-contained classroom through high school—and over two decades attending to the issues of technology in mathematics learning. Internationally recognized as a leader in both mathematics education and uses of technology in education, he is a Senior Scientist at Education Development Center, Inc., in Newton, Massachusetts.

Acknowledgements

I gratefully acknowledge the very substantial contributions of my EDC colleagues Ilene Kantrov, Sara Kennedy, June Mark, Deborah Spencer, and Kristin Winkler, who helped plan, think through, and critique the paper. Sara also did the library research about current curricular uses and prior studies of the effects of technology in mathematics learning. Thank you! I am also grateful for ideas that reviewers Andee Rubin and Kenneth Labuskes provided. This work was supported in part by the National Science Foundation Grant No. ESI-9617783. Opinions and views remain my own, and not necessarily those of my colleagues, reviewers, or the Foundation.