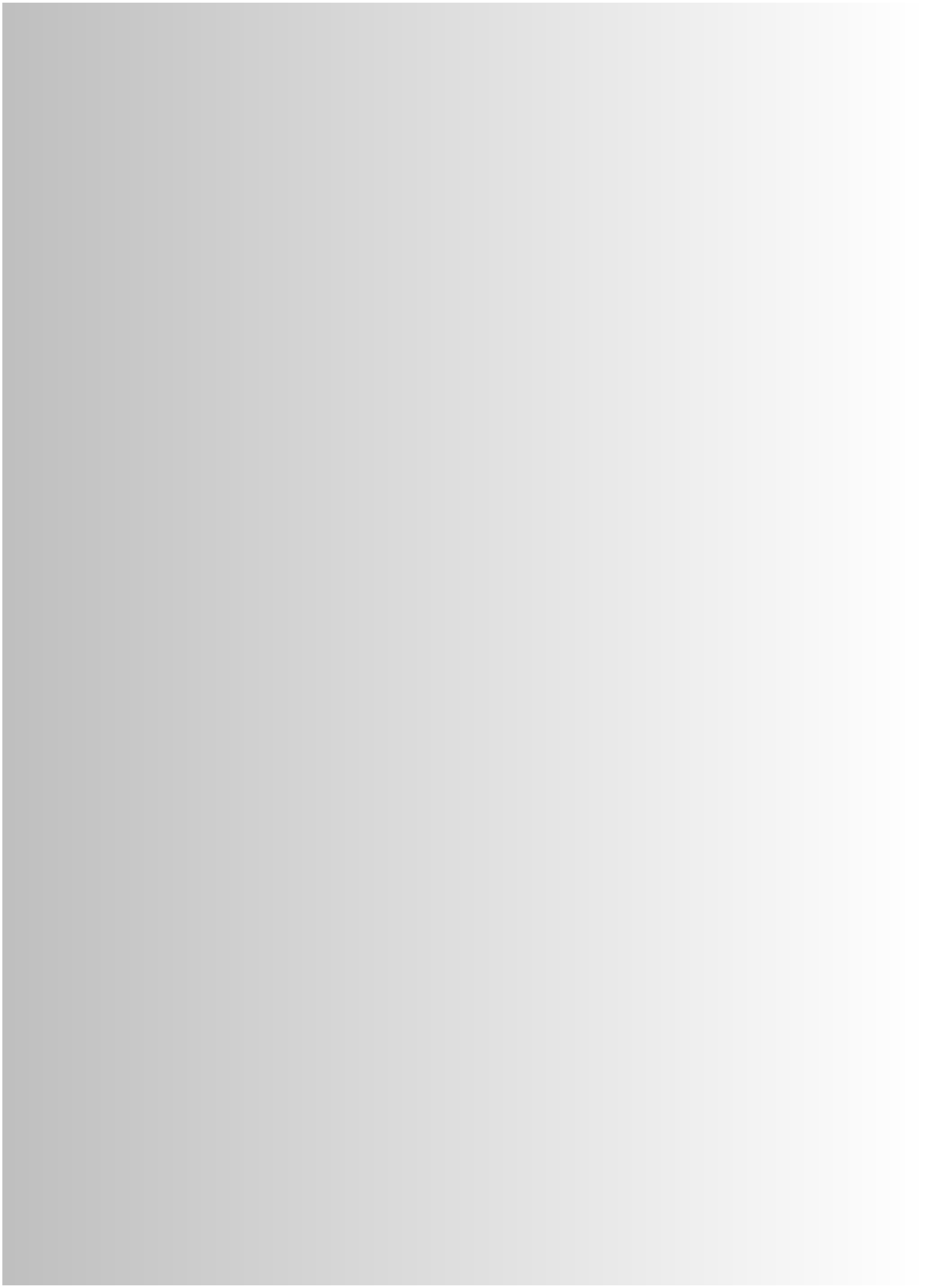
CONTEMPORARY MATHEMATICS IN CONTEXT



CONTEMPORARY MATHEMATICS IN CONTEXT

A high school curriculum for grades 9–12, developed by the Core-Plus Mathematics Project.

Contemporary Mathematics in Context is a four-year, integrated mathematics program that includes a three-year core curriculum for all students, plus a flexible fourth-year course that continues the preparation of students for college mathematics. The curriculum builds on the theme of mathematics as sense making.

Each course advances students' mathematical understanding along interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of these strands is developed within focused units connected by fundamental themes, common topics, and mathematical ways of thinking. The program emphasizes mathematical modeling and modeling concepts, including data collection, representation, interpretation, prediction, and simulation. Graphing calculators are assumed and appropriately used throughout the curriculum.

Each of Courses 1–3 consists of seven units and a capstone experience. Each unit has three to six multi-day lessons which develop major ideas through investigations of applied problems. The time needed to complete units varies from four to six weeks. The final element of each course, the Capstone, is a thematic, two-week, project-oriented activity that enables students to synthesize and apply the important mathematical concepts and methods developed in the course. Course 4 consists of 10 units that permit tailoring of courses to various undergraduate programs.

Lessons are organized in a four-phase cycle: Launch—a whole-class discussion of a real-world situation establishing a context for the lesson; Explore—small-group investigations of more focused problems; Share and Summarize—a whole-class discussion enabling groups to summarize results of investigations and construct a shared understanding of important concepts, methods and approaches; and Apply—a task to be completed individually to assess levels of student understanding.

In addition to the classroom investigations, the program provides sets of MORE tasks, which engage students in Modeling with, Organizing, Reflecting on, and Extending their mathematical understanding. These tasks are intended for individual work outside of class. The program also includes *Reference and Practice* student handbooks for reviewing and polishing mathematics students encountered in the previous courses.

Student materials for each of Courses 1–4 are available in two parts, A and B, both in hardcover. For each course, there are also two hardcover *Teacher's Guides* that correspond to the students' texts. The *Teacher's Guides* provide for each unit: overviews, background on the mathematical content, objectives, instructional notes and suggestions for promoting student investigation and collaborative work, solutions, and possible student responses. Also available is an *Implementation Guide* that supports teachers in using the program effectively.

Other teacher support materials include Assessment Resources (which include quizzes, exams, projects for each unit, and cumulative assessments) and Teaching Resources (which include blackline masters that support classroom activities) for each course. Assessment and Maintenance Builder CD-ROMs provide for customization of assessments and maintenance exercise sets for each of Courses 1–3. Downloadable software for the TI-82, TI-83, TI-89 and TI-92 graphing calculators supports student exploration, and is required for some investigations.

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CHRIS HIRSCH ▸ DEVELOPER

Chris Hirsch is a Professor in the Department of Mathematics at Western Michigan University, where he teaches both undergraduate and graduate courses in mathematics and in mathematics education. He is Co-director of the *Core-Plus Mathematics Project* along with Arthur Coxford (University of Michigan), James Fey (University of Maryland), and Harold Schoen (University of Iowa). Chris was a member of the first National Council of Teachers of Mathematics Commission on Standards for School Mathematics and chaired the Curriculum Working Group for Grades 9–12. He is a former member of the Board of Directors of the National Council of Teachers of Mathematics and the Board of Directors of the School Science and Mathematics Association.

Research influences on the development of the program

Development of the *Core-Plus Mathematics Project* (CPMP) curriculum, now published as *Contemporary Mathematics in Context*, was informed by research on teaching and learning and the NCTM *Curriculum and Evaluation Standards*. It was also influenced by earlier innovative curriculum development in which the authors had been involved.[1] We agreed on several overriding design principles, perhaps the most fundamental of which is that mathematics is a vibrant and broadly useful subject that can best be learned and understood as an active science of patterns.[2] So ideas of experimentation, data analysis, seeking and verifying patterns, and prediction are pervasive in the CPMP curriculum.

A second principle that influenced our work is the use of problems as a context for developing student understanding of mathematics. In the curriculum, the learning of mathematics is situated within the context of investigating and making sense out of rich applied problem situations.[3] Because real-world situations and problems often involve data, shape, change, and chance, we chose to develop the curriculum for each year along interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. This decision was also based on research that suggests that deep understanding of mathematical ideas includes connections among related concepts and procedures, within mathematics and to the real world.[4]

Another principle that we've held strongly is that in any attempt to develop a new curriculum, each part of the curriculum should be justified on its own merits. While we didn't ignore the sequential nature of mathematics, we often debated and thought hard about what is the most important and most broadly useful mathematics for high school students to learn today. In designing a particular course, we considered carefully the question, "If this is the last mathematics students will have the opportunity to study, is the most important mathematics included?" In that sense, the CPMP curriculum was developed from the ground up, as opposed to just being driven by preparation for future course-taking.

We have tried to design a curriculum that could reshape not only the mathematics that students had the opportunity to learn, but also impact the manner in which that learning would occur. Research suggests that classroom cultures of sense-making shape students' understanding of the nature of mathematics, as well as the ways in which they use mathematics.[5] A pervasive theme throughout the CPMP curriculum is the notion of mathematics as sense-making. Investigations of real-life contexts lead to discovery of mathematical concepts and methods that make

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[1] In addition to CPMP Co-directors Hirsch, Coxford, Fey, and Schoen, other principal curriculum developers are Gail Burrill (University of Wisconsin-Madison), Eric Hart (Western Michigan University), Brian Keller (Michigan State University), and Ann Watkins (California State University-Northridge).

[2] Steen, L. A. (Ed.). (1990). *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, D. C.: National Academy Press.

[3] Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25 (4), 12–21.

[4] Skemp, R. R. (1987). *The Psychology of Learning Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

[5] Resnick, L. B. (1987). *Education and Learning to Think*. Committee on Mathematics, Science and Technology Education, Commission on Behavioral and Social Sciences and Education. National Research Council. Washington, DC: National Academy Press.

sense to students and also develop habits of mind for approaching new problem situations in sense-making ways.

The curriculum is written to promote the use of small-group collaborative learning in addition to teacher-led class discussion that launches and summarizes investigative work. The notion of collaborative group work was inspired, in part, by the increasing use of project teams in business and industry. It is also based on theories about the importance of social interaction in developing shared mathematical understandings and the role of communication in the construction of mathematical ideas.[6] There is some evidence that small-group collaborative learning encourages a variety of social skills conducive to the learning styles of groups that are currently underrepresented in mathematics.[7]

A unified curriculum

The CPMP curriculum is integrated in the sense that each course features interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. But in addition, we have tried to unify the curriculum through attention to common topics that cut across strands, like symmetry, function, matrices, recursion, and data analysis. Symmetry, for example, is not only an important characteristic of geometric shape, but also helps one better understand data distributions and the behavior of functions.

We have also focused on mathematical habits of mind such as visual thinking, searching for and describing patterns, and making, checking, and proving conjectures as a means of unifying the strands. More globally the strands are unified by recurring themes such as data, representation, shape, and change.

Approach to algebra

Traditionally the focus of school algebra has been on techniques and skills for by-hand manipulation of symbolic expressions. This approach to algebra for *all* students was driven by the goal of preparing *some* students for calculus as it has been traditionally conceived. The prevailing view seemed to be that proficiency with symbolic manipulation was a prerequisite to conceptual understanding, application, mathematical modeling, and problem solving. Computing technology offers new possibilities for priorities and sequencing of algebra, so the study of algebra can now begin with the most natural and motivating aspects of mathematics—its applications.

In the CPMP curriculum, the primary focus of algebra is on developing students' abilities to recognize, represent, and solve problems involving relations among quantitative variables. The use of functions as mathematical models is a central idea in the curriculum. Concepts and methods of algebra arise naturally in the context of reasoning with symbolic rules for function models. This approach is based on earlier promising work on computer intensive algebra by James Fey and his colleagues at the University of Maryland.

For each family of functions (linear, exponential, power, periodic, logarithmic, polynomial, and rational), students investigate real-world contexts in which data

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[6] Cobb, P. (1995). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23 (7), 13–20.

[7] Oakes, J. (1990). Opportunities, achievement, and choice: Women and minority students in science and mathematics. In C.B. Cozden (Ed.). *Review of Research in Education*, 16. Washington, DC: American Education Research Association.

...compared to students in more conventional curricula, CPMP students are better able to formulate mathematical models for problem situations and better able to interpret algebraic representations and calculations.

patterns or problem conditions involve that type of function model. They analyze the mathematical properties of each function model. For example, in Unit 2 of Course 1, the emphasis is first on recognizing different patterns of change in graphs and tables and how those patterns can be represented either by function rules or by recursive descriptions. Then, in Unit 3, students study the family of linear functions in terms of the kinds of problem situations they can model and their general characteristics. Exponential functions are treated similarly in Unit 6. A somewhat unique part of the curriculum is the early introduction of recursion as a means for describing change from one stage to the next. This "Now-Next" way of thinking has become a bedrock for algebraic thinking that's woven through each of the CPMP courses.

The development of algebra through functions continues in the second course by studying power functions and then more generally quadratic polynomial functions. In Course 2 and subsequent courses, the algebra becomes increasingly intertwined with coordinate methods for geometry, with trigonometry, and with matrix models. In Course 3, the curriculum revisits function families with special attention to writing equivalent symbolic forms of function rules and to methods for rewriting expressions in equivalent forms, including solving equations, inequalities, and systems of equations. At this stage, students should be skillful at modeling and solving problems by reasoning with tables, graphs, and equations (or inequalities) and using connections among these representations. Course 4 extends the work with functions to include logarithmic, polynomial, and rational functions and the conceptual underpinnings of calculus, such as rate of change and accumulated change.

Student outcome data and teacher feedback from field testing of Courses 1–3 indicate that, compared to students in more conventional curricula, CPMP students are better able to formulate mathematical models for problem situations and better able to interpret algebraic representations and calculations. Some CPMP students were not as proficient at manipulation of symbolic expressions as comparison students, but they had learned a variety of technology-based strategies for accomplishing the same goals. We are finding that some teachers and some parents would prefer not to give up as much of the manipulation skills and practice to achieve the higher-level goals. As a result, the project made adjustments both in Course 4 and in the published versions of Courses 1–3. We have also developed *Reference and Practice* books for each of the first three courses to provide additional practice with algebraic skills.

Developing other mathematical concepts

Statistics, probability, and discrete mathematics are prominent in each year of the CPMP curriculum. More than one-third of each of Courses 1–3 is devoted to the study of topics in these strands. This is consistent with the NCTM *Standards* and the project's answer to the question, "If this were the last course that a student would take in mathematics, what is the most important mathematics that we would like them to study?"

Design of the geometry strand has been a particular challenge for us. The historical view of school geometry as a place where students learn about deductive reasoning in the context of axioms, theorems, and proofs—essentially organizing the teaching of geometry around an axiomatic system—didn't seem appropriate. Since we had committed ourselves to developing a curriculum that showed the broad utility of mathematics, treating geometry that way would have been inconsistent with the modeling approach that we took with the other strands. The challenge was to devise a coherent geometry strand which focused on the usefulness of the content without ignoring its logical structure. In Course 3, local axiomatics are used to focus on reasoning and proof in the context of parallel lines, congruence, and similarity. Here the integrated nature of the curriculum provides for economy of treatment. For example, the Law of Cosines and the Law of Sines developed earlier in the algebra strand are used to establish basic theorems on similarity of triangles. Conditions for congruence of triangles are then studied as special cases of the similarity theorem. Development of reasoning and proof skills is not restricted to the geometry strand, but is also included in each of the other strands of the curriculum. In Course 4, concepts and methods of geometry, algebra, and trigonometry are meshed in the development of models for describing and analyzing motion in two-dimensional space and surfaces in three-dimensional space.

Concept development and skills practice

Concepts and skills are both important. We have tried to take a balanced approach in which concepts come first and complement skills. Investigations of rich, applied problem contexts enable students to develop conceptual understanding of important mathematical ideas. Particular skills are introduced following conceptual development. There is some practice with those skills concurrent with continuing to emphasize problem-solving and mathematical modeling.

When you look at a CPMP text, you'll notice that the exercise sets for homework are organized around ways of using and thinking about mathematics. Following investigations, there is a set of additional tasks called MORE tasks—Modeling, Organizing, Reflecting and Extending. Some of these tasks provide students with additional opportunities to use the mathematics developed in the investigations to Model other real-world situations. Organizing tasks are intended to have students step back from real-world contexts and analyze the underlying mathematical structures and their properties more carefully. Within the Organizing tasks, there is skill practice with the operations and techniques that are part of those structures. Reflecting tasks provide opportunities for students to reflect on how they're thinking about the mathematics they've encountered, or how that mathematics appears in other school subjects or in their everyday life. Finally, there are Extending tasks which enable the better students to be pushed even deeper or further with their mathematical understandings. Some of the Extending tasks are complex, real-world problem situations that require more involved mathematical modeling. Others look at the abstract nature of the mathematical models in terms of symbolic reasoning and more complex symbolic manipulation.

The attention to by-hand symbolic manipulation skills increases with each course, as students mature mathematically. Special attention is paid in Course 4 to completing the development of all algebra skills that are necessary as preparation for calculus and to practicing the skills needed for college placement tests. Although there is not an extensive amount of by-hand manipulative practice in the student texts, what's there is sufficient for some students. We have found that for other students, it seems not to have been sufficient. The project has recently completed development of *Reference and Practice* (RAP) books that accompany each of Courses 1–3 so that teachers can elect to distribute additional manipulative skill practice across the school year. We see these as handbooks students can take home for additional practice as needed. These are not typical drill-and-practice books. If you look, for example, at the *Reference and Practice* book for Course 2, the first part of the handbook actually reviews and summarizes material from Course 1. The sec-

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ond and third parts of the Course 2 RAP book contain exercise sets for polishing skills from Course 1 and Course 2, with an emphasis on Course 1. The *Reference and Practice* books for other courses are similarly organized. Because of their review and summary nature, the RAP books can also be used by parents wishing to help their sons or daughters.

Standardized testing

Each CPMP course is the product of a four-year research, development, and evaluation cycle involving over 35 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas.

As part of the year-by-year field testing of the curriculum, we administered several standardized tests to CPMP students and to comparable students in more traditional curricula in the same schools. One of the tests is the "Ability to do Quantitative Thinking" subtest of the Iowa Tests of Educational Development, which is a standardized achievement test used by many districts to judge how well students are progressing in high school. We chose this test because one of the things that we are committed to is raising the bar for all students to be able to think mathematically. At the end of Course 1 and at the end of Course 2, CPMP students outperformed comparison students on the end-of-year forms of this test.

As part of the field test of Course 3, we administered an end-of-the-year test comprised of selected items from the National Assessment of Educational Progress (NAEP). CPMP students performed well above the NAEP's 12-grade national sample across all content and process categories. The project also monitored the performance of CPMP students on ACT and SAT college admission tests. CPMP students did as well as comparable students in more traditional curricula. I think it's fair to say that if you take an individual CPMP student who takes both the ACT and SAT, he or she is more likely to perform better on the SAT than on the ACT, because the SAT is focused more on mathematical reasoning and less on specific traditional mathematics topics.

On a large midwestern university's mathematics department placement exam—based on items from the Mathematical Association of America placement test program—CPMP Course 4 students performed as well as comparable students in traditional precalculus courses on algebra and advanced algebra subtests, and they performed better on the calculus readiness test.

The CPMP program itself emphasizes curriculum-embedded assessment and performance assessments. Because multiple-choice formats are used almost exclusively on standardized tests and mathematics placement tests, we have included a section on "Practicing for Standardized Tests" in the *Reference and Practice* handbooks. The materials provide students with opportunities to practice taking multiple-choice tests and introduce them to effective test-taking strategies.

Grouping students with different abilities

The CPMP curriculum has been tested and is currently being implemented in both homogeneously-grouped classrooms and in heterogeneously-grouped classrooms. In heterogeneous classrooms, the investigative nature of the curriculum permits different students to approach problems at different levels of sophistication. We have found that there are differences in the ways students think about problem situations and in the ways they prefer to approach problems. Some students, at least initially, prefer to think about situations numerically, others prefer to think about them more graphically, and yet others prefer a symbolic approach. Verbal, numerical, graphical, and symbolic representations and connections among them are developed throughout the curriculum. These multiple representations, often technology-based, provide more students access to important mathematics and to more complex problem situations. Students of differing ability levels achieve different depths of understanding as they complete investigations. In heterogeneous classrooms, they also learn new ways of thinking from their peers. Extending tasks built into each lesson are intended to challenge the most able students.

Technology

We have held to the principle that both computer and calculator technology continue to change what mathematics is important. These technologies also influence how mathematics can be taught and how students might learn mathematics. We've tried to build a high school curriculum for all students, not just students in affluent suburban districts, so we've been very careful about the technology assumptions of the curriculum. As a result, the first three years of the curriculum assume only graphing calculator technology. However, the project developed downloadable calculator software, including a PERT program[8], a geometry explorer, and a spreadsheet. Course 4 assumes students will have some access to computers. Schools that have access to computer labs can, of course, use computers with the entire curriculum, but we really did not want to develop a curriculum that presumed that students had regular access to computer technology. That position may change somewhat with a second edition.

When schools commit themselves to use of the CPMP curriculum, they commit themselves to the assumption that every student will have access to a graphing calculator or computer. Schools that find imaginative ways of making the technology available to students outside the classroom are able to proceed much further and much more smoothly through the curriculum.

Changing teachers' practice

Without question, one of the most common comments from teachers who have used the CPMP curriculum is that they could never go back to teaching the way they used to teach. We see that as one of the strongest possible benefits of the curriculum. As times change, curricula will also change. By the way we have organized the curriculum, by trying not only to focus on good mathematics but also on enabling teachers to teach mathematics in more active ways, we seem to have altered how teachers think about their practice.

We are finding that most CPMP teachers do believe that students can construct understanding out of problem situations, and that simply providing students with rules and recipes doesn't contribute to in-depth learning. Because of the investigative nature of the curriculum materials, and because we have written Checkpoints—opportunities for groups to share and summarize their findings—into the curriculum, we've provided natural places for discourse in the classroom. Teachers are thinking hard about the nature of that discourse as they reflect on the mathematical ideas that surface at the Checkpoints.

Outside observers who listen to teachers talk about their experiences teaching the curriculum have told us that the use of the curriculum seems to help teachers

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[8] PERT (Program Evaluation and Review Technique) is a discrete mathematics modeling tool that is used extensively by business and industry to schedule and help manage large projects. The CPMP developers created a calculator program that implements PERT. PERT software is also widely available for computer applications.

assess much better what individual students know and are able to do in mathematics. As teachers talk about students, they're now able to talk about individual students and what they do with particular tasks, as opposed to whether the student was simply an A student or a B student.

Overall, I think the use of the curriculum has strengthened and deepened teachers' understanding of mathematics. In our workshops, as we work with teachers who are preparing to teach a particular course for the first time, it is quite common to hear them make remarks such as, "Oh, I remember seeing this in college, but now I really understand it. In fact, now I know why you want to know it, because I see all of these applications." So, from a teaching point of view, CPMP teachers are not only beginning to understand that the teaching/learning enterprise can be different, but they are also developing a deeper understanding of mathematics and a broader understanding of its applications.

Support and professional development for teachers

The project encourages school districts to have at least one teacher in a building attend an in-depth summer workshop for the course level they're about to implement. Having a teacher study a course in-depth with other teachers from around the country—reflecting on what they're doing and sharing with other teachers not only the insights they're developing, but also some of the concerns that they have—is extremely important. The goal is for that teacher to go back and, as a lead teacher, work with colleagues during the summer before actual implementation. We recommend that, if possible, the same teacher repeat this process for each course in the curriculum.

We're finding that the lead teacher who attends a summer workshop often goes back and works at her district level complemented by a CPMP field-test teacher. For example, there's been a relatively large adoption of CPMP in a district outside Seattle this year. We had a curriculum coordinator and lead teachers here for a workshop this summer. They then went back and prepared and delivered a one-week workshop for all the teachers back at the site. They were accompanied by an experienced CPMP field-test teacher who had taught the course several times. The field-test teacher can bring in fresh ideas from the outside, and can speak to particular challenges of implementing CPMP.

Providing opportunities for teachers who are implementing the curriculum to interact with one another within a district also seems to be very important. The problems and challenges teachers face with implementation, and the ways they try to resolve them, are going to be somewhat unique to each particular collection of teachers. Finding ways that good, successful practices can be shared among teachers who are trying to make change is very helpful. We also try to do that through the CPMP listserv.

The project has been working hard to help our publisher understand the importance of professional development. We are at the point where the summer workshops are jointly planned and worked on with staff from the publisher. But perhaps an even stronger indication of the publisher's commitment to professional development is that they are providing customized, on-site staff development programs for school districts. Of course, this is somewhat determined by the size of the district. A cohort of interested CPMP field-test teachers have attended short institutes here on campus devoted to "teachers teaching teachers" during which they looked carefully at preparing one-week workshops that they, as teachers, can conduct with other teachers in local districts. The publisher and the project have been quite successful in connecting field-test teachers with districts that have adopted the curriculum.

Preparing for a successful implementation

In advance of a district using the CPMP curriculum, we encourage teachers who are involved to look very carefully at the program and at the needs of their student population. We also recommend they devote a considerable amount of time to helping parents understand why the school wants to change its mathematics curriculum and why they think the CPMP curriculum seems to be best suited for their students. We encourage schools who contact us about potentially using the program to study the curriculum very carefully and to consider questions like, "Will all mathematics teachers buy into the curriculum in terms of its content emphases? Will teachers buy into a more investigative approach to mathematics?" In contrast to past practice, changing to this kind of curriculum is more than simply replacing one textbook with a textbook that just has a newer copyright and perhaps a different organization. There are fundamental shifts in the mathematics that is emphasized and in the sequencing of mathematical topics. There are fundamental shifts in the kind of teaching and assessment expected with this curriculum. Careful analysis of the curricular content and building early support within the community are essential.

In considering any implementation, it's very important to address the needs of students who historically were accelerated or identified as students in honors classes. One of the biggest challenges we've found is when school districts simply implement Course 1 with all students in grade 9, without any provision for acceleration. That has led to difficulties with parents. We've found that schools have the most parent support when provisions are made to at least offer the possibility for accelerating a small group of students in Course 1 as 8th graders, and then provide opportunity for those students to continue on with each course in the sequence in subsequent years. That way, parents can see that the CPMP curriculum provides access to AP Statistics in grade 11, and access to AP Calculus in grade 12 or earlier. We have found that if schools build the groundwork for parent support, the curriculum can be successfully implemented with all students in grade 9 and with selected 8th graders.

A schedule for implementation

Based on the evidence the project has to date, we recommend that a school district begin implementation with a single course—Course 1—after carefully laying the necessary groundwork with all staff, including administrators and counselors, as well as parents. Then, we recommend that the teachers who have taught the first course move on to Course 2 in the next year, allowing the teachers to grow with the curriculum. While in a traditional curriculum a topic is taught and mastery of that topic is expected before the chapter is over, in the CPMP curriculum, progress toward mastering—developing a deep understanding of—a particular topic occurs throughout a course and across strands, and often continues in a subsequent course. To fully understand where the program is leading, teachers should grow with the curriculum through each year. My understanding is that this practice of teachers advancing along with students in a curriculum is common in other countries; Sweden, for example. A middle-school teacher who is teaching Course 1 to accelerated students would profit from attending extended workshops that engage them in exploring mathematical content from each year of the curriculum. ■

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TOMMY EADS ▸ MATHEMATICS TEACHER

Tommy Eads teaches *Contemporary Mathematics in Context* in the North Lamar Independent school district in Paris, Texas. North Lamar High School serves a rural population of approximately 900 students in grades 9 through 12. It was one of the field-test schools for Courses 1, 2, 3, and 4.

Tommy has been teaching *Contemporary Mathematics in Context* for five years, and has taught high school for 30 years. Before coming to North Lamar, he taught 7th- and 8th-grade math as well as high-school mathematics at a small rural high school. He has taught all four courses of CMIC.

Goals for students

When I attended the University of Denver for my Ph.D. in the '70s, I was privileged to have Dr. Ruth Hoffman for a teacher and advisor. She taught in a mathematics laboratory, so I saw how you can actually use groups and discovery learning effectively with teachers. Ruth led us to believe that we could also do that with students at the high school. When I came back and tried some of those things in my classroom, I was able to assess student understanding much better using working groups rather than with paper-and-pencil tasks. But I didn't totally buy into the notion until I started the *Core-Plus* program, because it really forced the issue.

My ideas of what students need to know when they graduate from high school have really changed. I used to see myself as being a math teacher who was preparing kids for college mathematics. While I think we did a fairly good job of that, we missed a lot of kids in the middle; there are a lot of kids who either don't finish college or never go. We want to reach those kinds of kids.

Campbell's Soup Company is the major employer in our city and also our major taxpayer. About seven years ago they sent a food tech engineer to speak to our math department about what we should teach that would help students to be better prepared for the world of work and also for college. She said, "Here are the things that we want kids to know. Number one, we want kids to be able to communicate clearly with complete sentences and concise paragraphs. Number two, we want kids to be able to work in groups to accomplish goals. Number three, we want people who will be able to do some mathematics on their feet, in their head. And when the mathematics is too cumbersome, we want people who know how to use the technology to solve problems, so that they have the wisdom to know when you can do it on your feet and when not, and then be able to use the tools as necessary." We were quite surprised, and that made us begin to look at what is it we needed to do to that end.

Why *Contemporary Mathematics in Context*?

What attracted us to this particular curriculum was that we felt it met our needs and that it would also prepare kids for college. The thing that had been very frustrating for me all through the years was kids who struggle with symbolic manipulation skills, primarily factoring. But there's a reordering in this curriculum of when and where the symbolic manipulation skills are expected. It's not existent that much in the first course, but it is important in the third course, and proof is important. In fact, if you look at all four years of this curriculum rather than just looking at each year as it stands alone, you will see clearly how the threads weave together. You will also see there's a fair amount of very abstract algebra in the third and fourth year. That appeals to me as a mathematics person.

If I'm talking to a parent whom I know has had a fairly good background in mathematics, I basically tell them that this four-year program is essentially what they had, except that symbolic manipulation skills are moved to Course 3 and are not in Course 1. In its place are such content items as some discrete math topics, some introductory statistics, and then also a three-dimensional approach to geometry. I've always felt that we start at the wrong end on geometry, that we start with two-dimensional stuff when we should be starting with three-dimensional stuff because that's the world our kids live in. So instead of starting with point, line, and plane, *Core-Plus* starts geometry with prisms and pyramids, and the kids are building them and exploring them and learning about surface area and volume. That's usually in Chapters 11 and 12 of the old textbooks.

The program does a really good job with the trigonometry. I like it because students are introduced to right-triangle trigonometry earlier—angular and linear velocity, transmission factors[1], things that are related to circular geometry that are important for kids to know—than we would have done in our traditional curriculum. The trigonometry aspect, which incorporates geometry, is brought in early.

Another thing that attracted me to this curriculum is that very little statistics was being taught at the high school level, and yet we kept hearing our students say, "We have to take statistics courses in college and we're not prepared for that." This curriculum infuses statistics; it's integrated throughout all four years. Students get a lot of work with variables there—usually what frightens a lot of adults when they take statistics are the formulas with all those variables with little bars and hats—but these kids are exposed to that early.

In this program, the kids are exposed to some formulas and ideas much earlier; that's the reason for the *Plus* on the end of *Core*. For example, transmission factor as it relates to angular and linear velocity was not in the traditional precalculus curriculum. Kids are exposed to things like the prismoidal formula[2] when they're looking at volume in first-year *Core-Plus*; I never saw that formula until senior mathematics.

Instructional approach

I had a 7th-grade teacher who would throw out a problem situation for us to think about and we would only go into the textbook for the skills that we needed to address those problems. That's the way *Core-Plus* works—we have a Launch to the lesson that presents the problem situation; it tries to attract students' attention by making the problem something that's relevant to their world.

Then, after the Launch there are usually some questions in the curriculum that cause the students to think about certain issues. These issues may be directed at teaching a new skill, or they may be directed at getting the kids to think about how to solve a particular kind of problem that may be imposed from a traditional curriculum.

After we discuss those issues without making any judgments about rightness or wrongness, we go into an Investigation, which is supposed to lead students to the objectives we want them to learn in the lesson. The Investigation may continue, or at that point we may have something called a Checkpoint. The Checkpoint basically raises questions that tie into the ones that came up in the Launch, but from a more mathematical point of view. Then the students are given an opportunity to do what we call an On Your Own, which is a short 15- to 20-minute guided practice of some of the things they're supposed to have learned.

After that, you will usually find a kind of summary: "This is what you were supposed to have learned." And then, after a couple of Investigations, we have what are called MOREs—Modeling, Organizing, Reflecting, and Extending activities

[1] Transmission factors are the relationships between the radii, diameters, or circumferences of two circles.

[2] The prismoidal formula is the formula for the volume of three-dimensional space-shapes.

...the challenge is being able to handle the questions that the kids are going to have, and, as the teacher, being able to ask the right questions to guide them in the direction they need to go.

that can be used for homework or class work or group work or projects. I tend to use the MOREs in the first year as homework, and then later on, once the kids get the hang of it, I begin to have them present some of those in class. So there are many opportunities for evaluation. The curriculum also has traditional things like lesson quizzes and unit quizzes.

Lesson preparation

There's a lot of preparation needed to teach this, and it's different from the old days, when preparation involved writing a lesson plan. With *Core-Plus*, the lesson plan is pretty much laid out in the book; the challenge is being able to handle the questions that the kids are going to have, and, as the teacher, being able to ask the right questions to guide them in the direction they need to go.

I used to spend time on lesson plans and tests designing items where the teacher would ask questions that would elicit correct responses from students. Today, the challenge is in phrasing your questions. It's not so much an issue anymore whether the kid gets the right answer; it's whether or not you asked the right questions. So being able to ask a question that will get them to see those differences is really tough.

Supplementing the curriculum

We're using the entire *Core-Plus* curriculum and occasionally supplementing it with some of our own things, mostly teacher-made tests and some other things, but not any other published material. When people are teaching in an integrated environment, they will tend to want to emphasize those subjects for which they feel a close relationship, particularly if they've taught them for many, many years. I've taught geometry for a long time, so I do supplement that to some extent with some of the activities and things that I did in a traditional classroom.

I also feel that there are symbolic skills that some of our kids still need at the 9th and 10th grade, before they take some of the standardized assessments. We have the freedom to do that—we don't feel like we have to stick right with the curriculum. So I tend to show them a little bit about factoring and show them some symbolic skills early on as the opportunity presents itself.

Technology

I also am a T^3 instructor—Teachers Teaching with Technology—with Texas Instruments, and we've been using the graphing calculator in our classes since 1988. There had been a lot of attempts to use technology in the classroom before we found this curriculum, but it was almost like we were overlaying the technology on the existing curriculum rather than using it in a meaningful way with the curriculum. This curriculum does use technology meaningfully, and I think some of the other reform curriculums do, as well.

In the '70s, Ruth Hoffman told us that as time goes on, there would be students in our classrooms who would know more about computers than we do, and we'd need to let them share their expertise. Sometimes with this curriculum, students who may not be the strongest math students but who do know something about the technology are able to share their ideas in the context of a mathematics classroom.

Implementation

Before *Core-Plus*, we had the traditional Algebra I/Geometry/Algebra II sequence, and we still have that track for those people who want it. We started *Core-Plus* with two teachers, myself and another, going to Western Michigan for two weeks of training in the summer. Although most of our 8th graders had pretty much made their choices about which math classes to go into in the fall at high school, we did get an opportunity for the 8th-grade teachers to recommend students who might benefit from this program. We also had a parent meeting and told them about the program so parents were able to put their kids in this program.

We were going to try to start with 60 students and have four sections, but we had 65 students to begin with. These kids were mathematically at-risk, socially at-risk. I would say that 90% of the kids we had originally were in an at-risk category. To some extent, that has put a label on the course here—that it was for dummies. Even though these kids were not dummies and they were able to show their friends they weren't, it still got labeled as such. We've tried to overcome that by changing the name of the course, but I wish now that we had really pushed harder to get some of the kids who were in the "honors" program.

We're on a block schedule and we did notice a pattern with these kids; attendance was not a problem. They would show up on the days when they had *Core*, even though they might not want to show up for some of the other classes that day. Some of them would tell me, "I'm going to be here for that class."

In the second year of its existence here, we made the mistake of putting everybody into a *Core* class. Teachers really need a couple years of teaching the curriculum to get a feel for it, and I think we should have phased in the number of classes over a period of time, just like we did the teachers. The biggest challenge right up front was trying to grade all the written work that kids did. So we've each taken our own initiative in trying to manage that; most of us have worked our tails off. When you're reforming, almost everything you've done in the past is out the window, so you're trying to do something else and at the same time keep papers graded and maintain some semblance of family life at home. It's really tough, but our teachers have risen to the challenge.

Grouping

We have had heterogeneous grouping up until this year. This year, mostly because of a state thing and because parents want it, we have the opportunity for kids to take the course for honors credit. What we've done there is cut back on the number of some of the MOR problems and give them more of the Extending problems. That's the only thing we've changed. We still have classes that are very heterogeneous, but we have a few that are a little more homogeneous so they can have the opportunity to gain honors credit.

I really feel the curriculum can meet the needs of students at all different levels. I think that continues to be a challenge, as to how you get that to work, but I honestly believe that it does. That's one of the reasons we were attracted to the reform curriculum to begin with. A lot of our kids in the middle and below were being placed in a two-year Algebra class, and about all they were getting was rules for signed integers—how to multiply, add, subtract, and divide, and a few other things. They weren't getting much math content, mostly just rules. We felt like we needed to do more for them, things that they would need if and when they decided to go to college.

One thing that I think this curriculum is doing for mathematics teachers is making us talk about how much mathematics we've learned.

In retrospect, I wish we had had kids apply to take the class way back then. I think that might have given us a more heterogeneous grouping to start with and would probably have advertised that this does meet the needs of upper-end and lower-end students. I still believe that kids of all levels can work together in a classroom to learn.

Training for teachers

Our *Core-Plus* training was very intense. It was two weeks of 8- or 9-hour days and we actually went through every aspect of the course. We were taught how to set up groups and how groups are supposed to work, and the trainers modeled this by example. We actually went through investigations on material that was not familiar to the typical 9th-grade teacher or first-year teacher. Then we also went through investigations of material that was familiar but yet had a different approach. That was extremely helpful.

We had homework every night, and I still believe that's important. Now, when I do training of teachers, I still emphasize that, "If you don't do it here you need to go back home and work through some of these problems, because they are different." So I think it's important for us to do what we're going to expect our kids to do.

Every math teacher in our department teaches a *Core-Plus* class. We started having other teachers trained, and now all of the math teachers in our school have been trained at least through Course 3. We felt that was important because every teacher needed to see the entirety of the curriculum for the first three years at least so that they would know the extent of what they were teaching. Ruth Hoffman always told us that every teacher needs to see that what they're teaching in kindergarten leads to calculus—when you're teaching partitioning to kindergarten kids, you have to know that that's a calculus skill. So what we do in training is look at statistics in Course 1 of *Core-Plus*, how that's going to be integrated into other things in Course 2 and Course 3, as well as how and where symbolic skills come in, so that we will all be educated and know. When we speak to parents, we'll say, "This is where they will have that."

We all use a lot of the *Core* things in the traditional classes we teach. We selected an algebra series this year that uses a lot of investigations. If you flip the book open, sometimes you can't tell much difference between the *Core-Plus* curriculum and this algebra book that we have because it uses a lot of the same things. Our teachers picked that intentionally.

Impact on teachers

One thing that I think this curriculum is doing for mathematics teachers is making us talk about how much mathematics we've learned. A lot of the mathematics that we know, we were taught by rote. We never really knew exactly why we were learning it, except we were just supposed to pass this on. For example, I now know why the quadratic formula works the way it does and how you can show it graphically, and I never knew that before. When I share that and symbolic kinds of algebra examples in this curriculum with other teachers, many of them will tell me also, "I've never seen anything like that before—that really makes sense."

In working with other teachers, I've found that the teachers who are willing to admit, "I don't know everything," seem to be the ones who are taking this curriculum and making it really work. Ruth Hoffman used to say that you will be a better teacher if you learn that it's okay to stand in front of your students and admit there's something you don't know. But if you do that, you have to be willing to accept the challenge to find out what it is you don't know and work with it and share that with your students.

Impact on students

Now I'm seeing kids who aren't afraid of fractions anymore, and they're not afraid to tackle word problems. I think we're all willing to accept the lack of some of the symbolic things in early *Core-Plus* as a tradeoff for the students' ability to do things with common and decimal fractions. That has been a weak area for the last 10 years in high-school mathematics. In this curriculum, there's a lot of reading, and that's a great strength. Our language arts teachers tell us they can see differences since our kids now have to read problems. Our science teachers are also really sold on this program. Almost no week goes by without one of them saying, "These kids may not know as much about symbolic skills—they all have weaknesses there—but they are not afraid to tackle problems, and they can really read graphs better."

The kids at our school who complete three years of *Core-Plus* have options for their fourth year. If they want to take a fourth year of math, they can take AP Statistics, or they can take Precalculus, or they can take Course 4 in *Core-Plus*. The students who come from a traditional curriculum typically have had Algebra in 8th grade and then they have Geometry, Algebra II and Precalc, and so their fourth-year option is mostly AP Statistics or AP Calculus or Math Applications, which is a state course.

While we don't have a large minority population, I do think this program has probably opened the door for more of our minority kids to take upper-level courses. In the past, it's been really difficult to get black and Hispanic kids to take upper-level math.

We've had a number of kids who are at-risk—I think we have 17 of them in college out of that 65—who I've tried to follow up on. The ones who go to our local junior college say that their math classes are "not anything like we've seen before, except we can do the word problems better than most of our friends." The symbolic algebra they feel deficient in; however, the ones who are willing to accept the challenge tell me, "Once we figure out what they want, we can do it." Now, some of them are quick to tell me that if they had had to do symbolic algebra as 9th graders, they didn't think they could have ever done it, plus they might have been turned off to math.

Special education

As far as content mastery of special education, that is still a little bit of a problem, because the reading level in this curriculum is pretty high. I know when I first started teaching it, I had kids who didn't really want to read out loud. I didn't force them to read out loud, but when you put them in small groups, they tended to be less apprehensive about reading. About midway through the year, I found that some of the kids who heretofore were not willing to read out loud in front of the whole class began to feel comfortable reading out loud. Some of our teachers who have large numbers of special education kids say they are finding the same to be true in their classes. What seems to slow those kids down is having to read for understanding and then solve the problems. I don't know how we're going to address that, but I think it's a concern everywhere, because there's so much reading and writing in this program.

Parents

Parents and other teachers who do not understand what discovery learning is all about do not understand this way of teaching, and so therefore they often think it's completely wrong. Trying to educate the public on that is very difficult. Last year we had five meetings at night for parents to come to if they had questions. While we advertised the meetings well, they were not very well-attended most of the time. Also, if you talk to the parents on the Internet or if you talk to them in large groups, sometimes there are people whose questions don't get answered because you have a dominating person who's there on a mission. I find it's better really to talk to parents individually. I know that takes more time, but for one thing I think they feel their concerns are better addressed.

When I have parents complain about *Core-Plus*, one of the things that I ask them is, "What did you like about Algebra in high school?" They will tell you, "Nothing," and they will be quick to say, "I hated it, I didn't like it." So it doesn't make sense to me that they want their kids to have to endure that which they hated.

Parents also worry about college admissions. We had one parent say, "At Texas A&M they don't accept *Core-Plus*." I said, "Did you tell them what courses your kid had?" "No, I told them we had *Core-Plus*." I showed them Saxon books or something like that and said, "What if we taught your child Saxon math? Then would you ask them if they accept Saxon? They'd probably give you the same answer, because the people in college don't even know what Saxon math is and they probably don't know what *Core-Plus* is, and they probably don't know what Addison-Wesley math is. But if you tell them, 'My child took statistics and algebra and geometry and trig,' then they'll understand that."

I've had a couple of parents who are working on education degrees over in Commerce, Texas, which is 40 miles away and has a branch of Texas A&M. One of them was really adamant against this program to begin with and she came back last year and said, "I looked at my daughter's *Core-Plus* book and that's what I'm having to take at A&M Commerce for my math preparation. Maybe you all are doing the right thing."

Our administrators have been supportive, and our assistant superintendent is a math person. She has fielded a lot of the questions for us, from some parents who only value the kind of mathematics that you can memorize, and that's all they think you're supposed to know. She goes to NCTM and she's been very involved at the state level with math reform. ■

BARBARA CRUCS ▸ MATHEMATICS TEACHER

Barbara Crucs teaches at Firestone High School in Akron, Ohio, where she was formerly the head of the math department. Currently, she serves as a half-day International Baccalaureate coordinator and half-day teacher of mathematics (*Core-Plus*) and Theory of Knowledge (IB). Barbara has taught at the high-school level for 11 years, and previously taught middle school for a total of 8 years.

Firestone High School has 1250 students, about 50% of whom are minorities. The Akron Public School District is a mid-sized urban district; it houses eight high schools. Teachers at Firestone High School refer to *Contemporary Mathematics in Context* as *Core*.

Goals for students

I want my students to be able to deal comfortably with issues that are more easily dealt with if you're educated mathematically. I also want to make sure they're well prepared to study at the university level in whatever level course is required for their majors.

I'd like to contribute to a generation of kids who, when they're parents, won't disable their own children from doing well in math because by saying that they never did well in math and that they never understood math. At some point, we need to change that pattern. One of the biggest strengths of the *Core* program is that it certainly helps students make sense of mathematics, and understand what mathematics really is.

Another thing I've seen that I think really points to *Core* as being so effective for the kids is something called "conceptualization at the point of utterance." This is the idea that you don't really understand something until you have to talk about it or write about it. That is such an integral part of this program for the kids. They are constantly writing while they're doing the investigation, and they're constantly talking about the mathematics, and one of the side benefits of that is what it does for their learning.

Why *Core-Plus*?

Five years ago, we realized that what we were doing at the time didn't seem to be meeting the needs of the students. The Ohio proficiency test was being implemented, and certainly the test scores were not as high as we felt they should be—in the system at large, not as many students were passing it as we felt should be. The administration downtown had heard of the program out of Western Michigan and applied on behalf of the district. When they read its objectives, and recognized the level of ability and professionalism among the writers and developers, it looked like a really valuable opportunity. We knew that the *Core-Plus* curriculum aligned with the NCTM *Standards*; we saw it as an opportunity to improve the math curriculum for the students. Our school and another school in the district were selected as field-test sites.

I was one of the field-test teachers and became the surviving field-test teacher at our school, so I have taught each level. We were among the first schools in the country that were trying to implement it in the school as a whole. Immediately, Akron Public Schools started teacher training programs in the summer using those of us who were trained at each level to then train other teachers in our system, with an intent to really implement across the whole school system.

This curriculum is incredibly strong, compared to what we were doing before. *Core* mathematics is rigorous, but, similar to the International Baccalaureate, it also uses the approach of involving the student and developing analytical, critical-thinking skills. If you see some of the TIMSS report and if you look at the videos where they're showing what's going on in successful classrooms around the world, it looks like the *Core* classroom. And if you look at what learning theory tells us has to happen in order for a student to learn, again it's the way that this material is being offered to the student. If you were teaching the old material with this type of approach, there would be better learning, but if you throw into that mix the fact that

...the content reflects the difference in the kind of mathematics that is important with the changing times.

you are getting mathematics that the student needs for today, it's just an incredible opportunity for the kids to learn and to grow and to be empowered mathematically.

The program is strong in directing the teacher according to the *Standards*, in terms of a different classroom organization—primarily involving the students much more directly in the learning process. We know how important that is in terms of helping ensure long-term learning rather than just short-term learning of a few skills without an idea of what to do with them afterwards.

Also, the content reflects the difference in the kind of mathematics that is important with the changing times. With the advent of inexpensive and increasingly powerful computers, there had to be a much bigger emphasis on probability and stats, and certainly discrete mathematics. *Core-Plus* does a far better job of preparing students for daily functioning, for just living. There's a much better understanding of what mathematics is. What students learn in math class is no longer isolated from the things that they see around them that require the use of mathematics or that have mathematics embedded within them.

Development of mathematics

One of the really nice things about the *Core* program is the materials, especially the support materials. There's really no need to supplement the materials; the things that you need are there. *Core-Plus* presents the mathematics in a different way so that in Year 1 there isn't the emphasis on mastery at the level that typically students were asked to do during their freshman year. But by the end of that third and fourth year, they've built up mastery layer by layer, and it's there. We are so impressed with the way that these materials are developed—not just the sequencing, but also the topics that are chosen and developed. We see things in this material that we didn't see in college and that are so important today, as well as new, really insightful ways of looking at the material.

For instance, the geometry was done from a dynamic standpoint, rather than a static standpoint. Triangles, for example, are looked at in terms of what happens to the properties of the triangle if you allow one side to constantly change in height. And you look at linkages so that you're studying the properties of quadrilaterals or parallelograms in terms of the dynamics of a rotating side.

These are different approaches, really, to traditional topics. There's an early lesson in geometry that deals with properties of polygons and builds understanding of how shape affects function. Students explore what happens when you increase the number of sides of a polygon in building a column. They consider the question, "How much additional weight could it hold if you increase the number of sides on the column?" Students are learning concepts that were the objects of particular lessons, but here they're learning them as an aside; the learning sort of almost sneaks up on them.

That's just one minor example. The kids love all the discrete math units where we're dealing with properties of networks. It's easily accessible and at the same time it's very obvious how powerful the applications of that tool would be.

The developers were very, very careful not to introduce names or technical things before the student is ready to use them; then these concepts are just assimilated into the lessons. One example that is used over and over again, is that students see rates of change and then see how that shows up in different forms, whether it's in a table of number values, or a graph, or real-world data, or an equation and where it's going to be in that equation. They're able to use that same concept to understand what's happening in some of the other types of patterns of relationships, like the quadratics or the higher-degree equations. It's really gratifying to see the ability of the student to understand the pattern of change—since really what we're looking for in mathematics is patterns. One of the things you see working in the program is those basic conceptual patterns being developed.

Instructional approach

When we started using *Core-Plus*, the group approach was much less familiar to my students than it is now. I had been using groups in my traditional classes prior to that, but not nearly to the level that I use them now with the *Core* program. For my Year 4 students, who have been in our *Core* program for four years at this point, group work is an integral part of the program. They are very accustomed to working in groups. It gives them a chance to discuss the mathematics and it involves them much more immediately with the material.

I know there were some teachers new to the classroom who had some trouble with classroom management because of the looser structure of the classroom. It's a different way of managing. You still certainly have to have control of the classroom, but you aren't as demanding in terms of making the students do specific things at specific moments as you would have been in a traditional classroom. It's a much friendlier, more nurturing kind of setting for the students. But you do have to establish the authority to make sure they are on task and that what they're doing is purposeful. Sometimes that's not easy.

Benefits for teachers and students

One thing that's going well is what teachers see—especially by the end of the second, third, or fourth year—in terms of what the kids are able to do. That was especially true for me when I had the students all four years while I was doing the field test. I really got a sense of where those students were at the beginning and how they would compare with comparable students in the traditional program. As a teacher you can see what the students are able to do, what they're able to remember to do and what they can tie together. You listen to what they're saying when they're working on the mathematics in the group, using those concepts and applying the math and being able to pull it back after not having used it for a while.

One thing that's going well is what teachers see—especially by the end of the second, third, or fourth year—in terms of what the kids are able to do.

And yet, getting acceptance of the curriculum is not easy. I have to tell you that we've fought long and hard battles, but in spite of that I wouldn't change a thing because it's just made such a difference for the kids in our classes. It's been worth all of it, and teachers like to learn. That's why I think we're in education, not just because we think it's important for kids, but because we believe in learning, and this is an opportunity for us to learn when we teach. Boy, I'll tell you, these materials are wonderful for that. It's just exciting to go through this material and see things from a new, more powerful perspective.

Testing and assessment

I see much less frustration in the *Core* classes than I used to in the traditional classrooms, because this curriculum really is empowering. I think the way the student will tackle tests is an example of that. They all know that there are things that they're going to be able to answer on that form of assessment. They're certainly seeing levels of mathematics that most students never see. So the fact that the students are exposed to high-level mathematics and can come to some understanding and mastery of it at a given level, tells me that it's working well.

The frustrating thing is for students not to be able to immediately see how far they've come with the curriculum. There's always been the problem of the student who struggles and who has to work very hard to achieve levels of mastery, but who has a quick willingness to blame it on the program. In *Core-Plus*, these students don't recognize that they are making a great deal of progress, compared to what they might have made in a normal classroom. However, some of them will speak to the fact that they always had trouble with mathematics and that finally there's a way of learning mathematics where they actually can learn it and feel some comfort with it.

Meeting different learning needs

One of the real strengths of this program is that it addresses different types of learning that students might not have access to in our traditional program. They have a chance to do so through investigations and calculator use in *Core-Plus*. Visual, verbal, hands-on, social, auditory, and other learning styles are all addressed through group investigations.

It's certainly more embracing of our ethnic groups. Our non-*Core* honors classes are much more homogeneous in color and in student types than our *Core* curriculum classes. We offer a much better product in the *Core* curriculum, and it's offered in a way that is accessible to students who might have been excluded because they hadn't "purchased the ticket" that got them into either Algebra in 8th grade or some other demonstration of some level of ability in mathematics. The real wonderful part about the *Core* curriculum is that students get exposed to this high level of mathematics and sophisticated topics while they still might have some weaknesses in terms of skill manipulation.

Implementation of *Core-Plus*

Our district mathematics program is really both the *Core-Plus* curriculum and also a traditional curriculum, with more students enrolled in the *Core* curriculum than in our traditional curriculum at this point. When we first decided to implement the *Core* program, we decided to allow those students who were in Algebra in 8th grade to continue with that direction, and that tended to be the higher-level student. Because we implemented the *Core* by requiring it of everyone else, there was opposition. There was strong opposition to being put into a program about which people didn't feel they knew enough. But the curriculum specialist downtown saw the advantage of this program and felt it was important for every student in the system to get this mathematics. We're still feeling the effects of that initial method of implementation because there's still a smoldering resistance to the program. When the program was made optional, it began to take over the curriculum; I think it could have done that for us from the beginning.

Because of the opposition, we found we had to offer the traditional track as well and give students the option of doing either. So we have kind of a strange assortment of students in the different classes. The kids who were accelerated in middle school are on an honors track that's traditional. Students whose parents weren't there to advocate for them weren't put into this honors program (which they believed was going to be a more rigorous program), so we have some very capable kids who didn't go that route, but who have ended up in the *Core* program.

Originally, we had the honors and the *Core* as being comparable in terms of level, and then we had a lower track for students who were really struggling with their mathematics. We've eliminated that because there was legislation passed that students have to have Algebra or higher in order to get a credit at the high school level. So we now offer an alternate traditional track that tends to be students who are not pushed as much mathematically.

Building community support

One thing that hurt our implementation of the program was initially saying that everyone was going to take the *Core* when few really understood the need for change. I think there needs to be education conducted by the school for the benefit of the community. At the same time, it's important to educate those reticent teachers, pointing out why there needs to be change. If they aren't familiar with the *Standards*, then reading some of the central issues that were addressed there and looking at the TIMSS report can help to underscore the need for change.

There was a lot of opposition to the program when it was first adopted, with concerns about students being used as "guinea pigs." There was concern that the math was not as rigorous as it would be in a traditional program. Although we did some public education—mostly for parents—prior to implementation, we had to continue it during the first and second year of implementation of the *Core* program.

Also, not every teacher was sold on trying new mathematics. It was something they didn't know, and you know how difficult it is for people to change. That certainly wasn't the case for teachers who were involved in a professional way in NCTM and who read current literature.

Now there are more and more results for people to look at, if they're really open to looking at them. I think that because we're letting *Core* ease its way in a little more now, there is more of the sense of letting it go and letting it do its thing and letting it prove itself. Teaching this material is more difficult for some teachers than what they've always done. For that reason, there is still some opposition within some of the schools in the system. I don't know how you get around that except by gradually hiring teachers who are more aware and better educated to what we need for today.

Transitions across grades

In terms of the transition from middle school, it's a very different mathematics when students get to the *Core*. At the first unit, it looks different to them. For years, they've done math where you practice the traditional 20 problems and you learn a new problem and then you practice that one. The growing confidence in the *Core* program facilitates that transition, but initially I don't think it was so easy.

The *Core* curriculum was developed with the idea that if a student took no more math than Year 1, what is the most important math for them to know; and if they took no more than Year 2, then what would be the most important math for them to know? So I think it does an incredible job of developing some sense of numeracy.

Year 4 of the program is designed specifically to help students function at the next level in college, whether it's a reform program or whether it's much more traditional. We've only had one group of *Core* students who have graduated, and the ones I've talked to have done well going on to that next level. ■

JAY NEWMAN ▸ DISTRICT SUPERINTENDENT

Jay Newman has been the Superintendent for the St. Joseph's County Intermediate School District in southern Michigan for one year. Prior to this, he was the assistant superintendent in Sturgis, Michigan for two years, the high school principal there for two years, and the assistant principal at the high school for four years. Earlier in his career, Jay spent 10 years teaching biology and chemistry in the Chicago area.

There are nine school districts in St. Joseph's County, two of which are using *Contemporary Mathematics in Context* in high school. Of those, Sturgis High School has been using *Core-Plus* for six years. The Sturgis School District serves about 3000 students and is changing dramatically in its demographics. In the past 10 years, the ESL population has grown from three students to 300 students; and the free and reduced lunch program has grown from 12% to 40% of students. The community is predominantly Caucasian, with about 9% Hispanic students, and smaller percentages of Asian and African-American students. Local and nearby automotive and machining factories and nutritional laboratories are major employers for residents of Sturgis and surrounding towns.

The call for change

In Sturgis, our Business Education Alliance was the wake-up call. After 18 months of soul-searching, the business people and the education people reached some common ground and agreed that the students who were graduating and going to work for the businesses in Sturgis did not have the mathematics background to do what had to be done at those companies. It became obvious that with the math they were taking at the time, about 60% of kids were graduating from Sturgis High School with Prealgebra as their most advanced math class. We also would run about four sections of General Math, which was a very basic math, and about half the kids were failing that. The success rate wasn't there.

One of the resulting major expectations of the Business Education Alliance was that kids would graduate from high school with the competencies necessary to be successful, if they went to work in a factory or in a business within the community. At the top of the list was that students would have the mathematical skills, of course, to be able to add, subtract, multiply, divide, so on and so forth. But they would also be able to deal with various types of measurements, and be able to do the rudiments of some statistical process control, which is required for all of the total quality programs that are going on. Almost every factory now has either ISO 9000[1] or whichever permutation they're on at this point, and in order for employees to fit into that, they have to understand some of these mathematical concepts.

Our belief was that our students' low interest in advanced math had a lot more to do with boredom than it did with the kids' ability to actually do the math. So our hope was to find a math program that met the needs of business and industry, that still prepared kids for the rigors of college mathematics, and that would stimulate those kids who had not been interested in math in the past.

Why *Core-Plus*?

Kathy Parkhurst, who's a teacher at Sturgis, had gone to a meeting of math teachers who were talking about the *Core-Plus* program. She saw some of the early materials and thought, "Boy, this is exactly what we're looking for." So she brought some copies back and shared them with other staff members at Sturgis. They said, "This is it! This is what we're looking for," because it truly was an integration of the mathematics principles. We knew that when students completed Course 1, they would have done some algebra, some geometry, some trigonometry, some discrete math. They would have been involved in statistics and probability. So Kathy contacted Chris Hirsch at Western Michigan and asked to be a part of the pilot. That's how it started.

In the traditional math sequence, I think students see mathematical concepts pretty much as unique, separate little entities, rather than being interrelated. When I worked in environmental assessment and needed to solve a particular scientific problem, I drew on all fields of mathematics to come up with solutions that could be used. A major strength of *Core-Plus* is that rather than teaching math as separate disciplines, like algebra, and geometry, it integrates those together, so that the student will take a look at the totality of math as being something used for solving problems.

[1] ISO 9000 is a set of industry standards for quality certification.

Prior to *Core-Plus* at Sturgis, only 30% of graduates ever had any geometry, and only 20% had trigonometry. Students would only get probability and statistics if they went to Advanced Math, which was precalculus. If you're teaching *Core-Plus* as the main mathematics curriculum, then all students, rather than only the elite students, are going to have experiences in algebra and geometry and trigonometry, etc.

One example is a problem that had to do with purchasing cars. Kids had to do some comparative shopping, actually taking ads from the newspaper and looking at what was available on each car, what the cost was, and so on. Then they would take a look at the relationship between cost and the various features that were available. They developed graphs based on the number of features and the cost, so they could take a look at linear comparisons. At the same time, they could look at fuel economy and the comparisons you could generate from the size of the engine and the amount of fuel economy. They were able to take one particular example and look at linear comparisons and geometric comparisons, because the variance in engine size and how the performance of the engine varied was not linear. And then they could also do some prediction of various features based on price.

I know that they also did a number of things with Euler circuits, in which they could do some comparisons of how you would use that particular system for planning a party or for driving from point A to point Z. So there's a lot of integration of everyday activities, and taking a look at one mathematical concept and how it can be used for a variety of different functions.

The incorporation of probability and statistics stands out, simply because most traditional programs don't even cover it. My daughter, who will be a junior next year, has been through *Core* 1 and 2, and she corrects me on some of my statistical things now. I think that's wonderful. Before we did the *Core-Plus* math, kids were not taught any statistics at all, other than a couple of little things in their biology or chemistry class, and now, every single kid understands mean, median, and mode, and they understand central tendencies, and they know standard deviation. If they get through the second course and start working into the third course, they start understanding analysis of variants and can tell you correlation coefficients. They can talk about a whole raft of different statistical maneuvers and tell you when you're using the wrong one. They will make comments about TV commercials where statistics are quoted; kids will say, "Hey, that's totally wrong." I think that's really fun to see the kids understand and be aware; they're more educated consumers.

Implementation

We made a commitment right at the beginning that we were going to go for a complete changeover, and that every incoming freshman, unless they were advance-placed into Geometry, would start right off with the *Core-Plus* 1 program. And then the second year, all of those from *Core* 1 would go into *Core* 2. The sophomores who were in the advanced track would continue on in the traditional program. And so we added each year as materials were available from the *Core-Plus* office at Western Michigan. The program made such a profound difference, almost immediately, in how the kids were doing in math. Kids who were traditionally failing math left and right were now being successful. We saw our freshman failure rate drop from about one in three down to about one in 10 in the first year. We liked that.

One of the most important things in implementing *Core-Plus*, I think, is to go after it 100% and do the whole thing, rather than trying to piece it in. I've seen a number of districts where they've run traditional programs and the reform program

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simultaneously, and it creates some real problems. The schools that I've talked to where they've run two parallel programs tend to get kids wanting to bail out of the *Core-Plus* program, because they see their friends over in the Algebra class doing half the work the *Core-Plus* kids are doing. There's a lot of work required in order to be successful in the *Core-Plus* program, and it takes a long time to do some things. In traditional programs, kids can sit down and do a bunch of problems, most of which are almost identical. With *Core-Plus*, they have to think every step of the way and utilize their high-order thinking skills. They have to do things that they're not used to doing.

Impact on students

In Michigan, we have our Michigan Educational Assessment Program (MEAP) test. The mathematics portion of that test is written so that students aren't just doing computation, or just solving problems; they actually have to explain the process that they go through in order to come up with the solution to a problem. They have to write about their mathematics. And because the students do that in the *Core-Plus* program, when it came time for students to start taking the High School Proficiency Test, which is part of the MEAP, we saw a dramatic improvement. Before going to the *Core-Plus* program, we had about 27% of our students passing the High School Proficiency Test in math. The very first year that we had students taking the Proficiency Test who had also been in the *Core* math for at least two years, it jumped to over 50%. And I think the most recent statistics are showing that close to 70% of the students are passing that. When I say pass the test, that's getting a state endorsement, which says that you are proficient in mathematics.

We've had some tremendous reports from kids who go on to college, and who have been highly successful in college mathematics. A young lady who struggled through the *Core-Plus* class in order to get B's in high school went to St. Mary's College—which is affiliated with Notre Dame in South Bend, Indiana—and got an A in her calculus class. So we know that it is preparing kids fairly well. I have heard from some kids who said that Calculus was really hard in college, but these were kids who had talent but didn't really work as hard as they should have in high school. We've had some really nice letters from kids who were in college and said, "Boy, this course was exactly what we needed."

And a real positive that we have gotten all along is that the science teachers have said that they can now teach more science, because mathematical concepts, like how to do charts and tables, how to do some statistics, how to organize yourself, and how to discriminate between good and irrelevant data, are now being taught in math.

Parents

It's very important to get parents involved right from the start. One of the arguments that we constantly heard after we changed over to the *Core-Plus*, was "I can't help my kid with math." Of course, many parents probably couldn't help their kids with algebra, either, because they couldn't remember it or couldn't do it. So one of the things we did was hold parent meetings, and actually gave them some of the activities from the math program. Teachers would teach the parents how to do these activities, so that they would become familiar with the approach. And those parents who knew what skills people need in order to be successful in business and industry really loved this right off the bat. Some of the people who had difficulty with it were some of the parents who are engineers or doctors, for whom the traditional approach to math was "fine for me." But one of the nice things the teachers would do is say, "Okay, how many times do you sit down in your job and just say, 'Now I'm going to do algebra' or 'Now I'm going to do geometry'? When you have to use your math, do you really think about whether it's algebra, geometry, trigonometry, of whatever? You just use it. And fortunately for you, someplace you learned how to integrate these things into your function—probably not in a math class, but probably from the science class that you took. So why not teach that to the kids in math?"

Professional development

Before teachers could teach the course, they needed to go to a training session in which they went through the entire course in two weeks. It wasn't just for the content, but also for the pedagogical techniques that are needed in order to successfully teach this. This isn't a course where you lecture. You may do some mini-lectures, and some setup for the kids, but the kids need to do all the work. What the teacher needed to do was transition from being the lecturer to being the facilitator of learning. And that took some real training on the part of teachers, especially those who've been around for a while and who had been teaching in a very traditional way.

Probably the biggest transition was making sure that we used more of a buddy or mentor system, with the more veteran teachers within *Core-Plus* serving as mentors for the ones who were just coming on. We tried to make sure that people who were teaching the same class had similar planning periods so that they could collaborate and discuss things.

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Changes for teachers

During the first implementation phases, the first time teachers were teaching a course, they would do a lot of comparison grading. For instance, teacher "buddies" would say, "Okay, first you grade your papers, and I'll grade my papers, and we'll swap them and see if we agree on how we've graded these." Grading *Core-Plus* work takes a more subjective evaluation, and there's a tremendous potential for a large variance on how things are done. We've tried to standardize some of the grading by making sure that people compared notes on those things. Teachers moved from a traditional grading system to a scoring rubric. They use scoring rubrics where a five means that you did absolutely everything correct throughout the whole assignment or test. A four on a particular problem would mean that you got everything right but maybe you didn't explain it quite as well, or maybe you explained it very well but there was an error in your logic someplace. And so on. In addition to creating a great deal of camaraderie within the department, using and understanding the scoring rubric helped math teachers become mentors for teachers in other departments; rubrics were something that we were trying to bring to other departments within the school.

Before *Core-Plus*, we had the person who taught Calculus and Precalculus, and the person who taught Advanced Algebra/Trig, and the Geometry person, and the Algebra person, and the Basic Math people. No one really talked to one another because they did separate things. Once the *Core-Plus* came in, they had some real common experiences that they would discuss together. During the school year, the teachers themselves get together in regular meetings to discuss what they are doing, especially if they are teaching the exact same class or exact same course. They get together to make sure that they are on the same page, and going through things in the same way.

There were teachers who were real hesitant, because there were things that they thought they would not be competent in, especially when it came to areas of discrete math, and some of the probability and statistics. Some of them had never really dealt with Euler circuits before, for instance. So, a lot of new topics had to be dealt with. But I think most of that was pretty much alleviated through the training. A few of the teachers didn't want to get involved in Course 4, the highest level course, but that wasn't a real problem.

Administrative challenges and support

In the middle of implementing *Core-Plus*, we went from a traditional schedule to a four-block schedule, so we transitioned from a year-long course to teaching the course in a semester. This brought up a lot of the issues like, "What's going to happen when kids have a whole semester off before they take their next math class?" or, "What if they take a whole year off before they have their next math class?" We've tried very hard to get counselors to make sure that no more than a semester gets in between the first course and second course of the program, and so on.

Since we were involved in the early stages of the pilot and they were writing the courses as we were going along, the printed material for the various units wasn't done when we were ready for it. We would get kind of a prototype, and then we would have to copy it ourselves. We incurred a lot of costs, as far as copying was concerned. We increased the supply budget, from around $3000 a year for math up to $12,000. We also gave Kathy Parkhurst a one-hour release time to serve as a resource person for the department. She would do the photocopying and all the emergency stuff. She also went around and talked to all the teachers, and made sure that they were on track and had what they needed, or gave them the help that they needed. So it was a major financial commitment that was made in order to get the system to work. Once you make the decision that this is an important thing to do and that this is a top priority, then you find the money. If you make a priority list from one to five, and if professional development is a five, you'll never find the money. If it's a one, you'll always find the money; even when the budget is extremely tight, you'll find other places to take it from. It's a priority issue. We rearranged things and we got a commitment from the district office to support this.

The Sturgis School District made a major commitment back when it first got started in this direction, as far as doing reformed mathematics. The high school is doing *Core-Plus*; the middle school is doing *Connected Math* out of Michigan State University. In fact, Sturgis has gotten to a point where they're doing a lot more pre-testing of kids, and accelerating kids right into *Core-Plus*, rather than having them do the 8th-grade course of *Connected Math*. We're trying to eliminate duplication of content as much as possible; I think the programs are working very well together.

Sturgis now has a new superintendent, a new assistant superintendent, a new business manager, and a new high school principal. This will be the fourth high school principal to work with *Core-Plus*. When we put the program in, I was still the assistant principal at the high school, and then I moved into the principalship. The superintendent at the time was very supportive of it, as was the director of instruction. And when I moved to be the assistant superintendent in Sturgis, I was very supportive. Now we have all kinds of new people who don't have history there. It's going to be very interesting, because one thing that this program needs is support from the administration. ■

