INVESTIGATIONS IN NUMBER, DATA AND SPACE

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An elementary school curriculum for grades K–5, developed by Investigations in Number, Data and Space at TERC.

The title of this program, *Investigations in Number, Data and Space*, reflects the view that mathematics in elementary school is more than arithmetic. In the elementary grades, students need to develop a foundation in several key content areas of mathematics: number, data, and geometry, or space. The *Investigations* curriculum also includes activities based on recent research on young children's understanding of the mathematics of change.

Each *Investigations* unit offers from two to eight weeks of mathematical work on topics in number, data analysis, and geometry; the number of units per year varies by grade level. Because of the many interconnections among mathematical ideas, units may revolve around two or three related areas—for example, addition and subtraction or geometry and fractions.

In each unit, students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas. Rather than working through a textbook or workbook doing page-by-page exercises, students actively engage with materials and with their peers to solve larger mathematical problems. Students use concrete materials and appropriate technology, including calculators, as a natural part of their work. They work in a variety of groupings: as a whole class, individually, in pairs, and in small groups.

What all the investigations have in common is students working in depth on a number of problems, actively using mathematical tools and consulting with peers as they find their own ways to solve the problems. The investigations allow significant time for students to think about the problems and to model, draw, write, and talk about their work. In addition to the investigations, the curriculum also includes games and classroom routines that support mathematical thinking.

Investigations looks quite different from a traditional elementary program. While it provides all the information teachers need to implement a complete mathematics curriculum, there are no student textbooks. The main teaching tool is a single teacher resource book, called a curriculum unit, for each unit in a grade level. Each curriculum unit provides lesson plans, materials lists, reproducible student sheets for activities and games, assessment activities, and other support materials for teaching the unit. Some units at each grade level include computer software. Optional *Student Activity* books provide the reproducible unit pages from the *Teacher Resource* book in a booklet for each student. *Investigations at Home* booklets for every unit are designed to provide extra practice and to inform and involve families in this kind of mathematics.

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SUSAN JO RUSSELL ▸ DEVELOPER

After 10 years of classroom teaching and staff development in elementary schools, Dr. Russell became involved in research and development. She is currently a senior staff member of the Education Research Collaborative at TERC, where she has been for 16 years. She directed the development of the *Investigations in Number, Data and Space* curriculum and co-directs a project that is creating professional development materials for elementary-grades teachers, *Developing Mathematical Ideas*. Her current work focuses on issues of professional development and leadership development in elementary mathematics and on understanding how practicing teachers can learn more about mathematics and about children's mathematical thinking.

Goals of *Investigations*

There were four major goals of our work in developing *Investigations*. One of those was to offer students meaningful mathematics problems. We believed that in order for students to have meaningful mathematical experiences, the problems themselves had to have lots of different entry points, ways for the whole range of students to enter those problems and do significant mathematical work. *Investigations* was conceived from the beginning as a curriculum that needed to meet a whole range of needs within a grade level. The problems were designed and tested with that in mind.

A second goal was to emphasize depth in mathematical thinking, rather than superficial exposure to a series of fragmented topics. There's more good mathematics at any grade level than you can possibly do in a year. We really had to think very carefully about what the important mathematical ideas are at each grade level. If part of our enterprise was to really engage students deeply in thinking about mathematical ideas, then we had to make some choices about what is the most important content at each grade level. We also tried to develop a balance of content across the grade levels. The focus of *Investigations* is not just on learning a set of skills and procedures and information, but is on developing the capacity and inclination to reason about mathematical ideas. That includes being able to describe one's own mathematical ideas, make connections between one's own ideas and somebody else's ideas, look at whether what you're saying is reasonable, and justify the mathematical statements that you're making.

We made a very deliberate decision to design the curriculum as a modular curriculum, so that at each grade level, it's organized into 8 to 12 modules, each of which focuses around a few big interrelated mathematical ideas. That decision came out of our belief that many of the ideas are complex, and that students need to have time to really think through the ideas in a variety of contexts, and to make connections among those ideas, rather than jumping from topic to topic.

A third goal we had was to communicate mathematics content and pedagogy to teachers. Teachers must be able to continue to learn mathematics content and also to learn about children's mathematical thinking as they teach. We really see *Investigations*—the curriculum itself—as a professional development tool, as well as a curriculum for students. Even if elementary school teachers were coming out of their pre-service teacher education prepared in mathematics—which is often not the case—but even if they were coming out prepared, their learning about both mathematics content and how children develop mathematical ideas would still have to be an ongoing, long-term process throughout their careers. You can't possibly learn everything you need to know in a pre-service setting before you have your own classroom. We felt that the curriculum itself had to provide materials from which a teacher could continue to learn as she uses it over multiple years. There is enough material in *Investigations* to help the teacher take a fresh look at what's happening in her classroom even in the fourth or fifth year of teaching it.

A critical guiding principle for us is that teachers' knowledge about what is important mathematics for the elementary grades, and their knowledge about how children learn mathematics at these grade levels, need to be intertwined, and need to inform each other. What children bring to the classroom, in terms of their ideas

about mathematics, is an important component of how teachers build their mathematics instruction. In order for us to develop our curriculum in a way that took into account both the important ideas of the discipline, and the ideas that children bring to the classroom, we as developers needed to know a lot about both the discipline and how children think about those ideas.

It was very important to us that what we saw in classrooms—what students did, how they talked about mathematics, what ideas they struggled with, and what kinds of connections they made among mathematical ideas—was used to give teachers glimpses into classrooms where these materials had been used. Teachers needed to have access within the curriculum to the kinds of thinking that they were likely to see in their classrooms.

Our fourth goal was to substantially expand the pool of mathematically literate students. There are many groups of students in this country who have been traditionally poorly served by the mathematics curriculum, and that includes students of color, girls, special education students, and language minority students. We worked toward that goal by making sure that the curriculum was tested in a wide range of classrooms: in both urban and suburban classrooms, in classrooms that were designated by the school as gifted, in classes that were designed by the school as special education, and in classes that had lots of diversity in terms of ethnicity and language. We wanted to make sure that our materials were able to engage the range of students in significant mathematical work.

The mathematics of *Investigations*

Our title is *Investigations in Number, Data and Space*. Number, data, and geometry were three emphases throughout the curriculum and throughout the grades. Traditionally in American education, the largest proportion of time in the elementary grades has been spent on number, and we think that number is still a very important topic, probably the most important topic within the elementary grades. But we also wanted to broaden what students had access to, in terms of different topics within the discipline of mathematics.

Number and operations is a central piece of our curriculum, and we spent a lot of time and effort developing it. We think it is particularly strong and well worked out. Our emphasis in the area of number is on helping kids to develop a sense of number and the magnitude of numbers and their relationships; an understanding of operations and their relationships, through a focus on the base-ten number system; a sense of developing benchmarks in that system; and a large repertoire of number relationships, including basic number combinations, or number facts. We want students to really develop an understanding about what the structure and characteristics of the operations are, what they model, and how they're related to each other.

When I say "benchmarks in the number system," I mean things like, "What is 372? What is its position in the number system?" It's not enough for kids to be able to say, "Well, that's three 100s, and seven 10s, and two 1s," which we know they can learn to say. What we want them to know is how that number is related to 370 and 400 and 350, and 3,720. We want them to build a model in their mind of the structure of this base-ten number system, in a way that they can use to solve problems.

We're very focused on the idea of developing computational fluency, not by learning a particular algorithm, but by developing algorithms that you can rely on, that you understand, and that are built on number knowledge, on knowledge of place

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value, and this repertoire of number relationships. We want that computational fluency to be grounded in not only students' understanding of the base-ten number system, but also in their understanding about the way the operations behave, what they're for, and how they're related. So, for example, the relationship of multiplication to division is something that's very important to understand in solving division problems. We also expect students to develop a large repertoire of number relationships that includes the basic number combinations of addition and multiplication, and the related subtraction and division combinations; what people call facts. There are also lots of other number relationships that are useful for students to know—like how many 25s are in a 100, and how many 100s are in 10,000, and so forth.

Data is also a strong strand throughout the grades. We don't spend as much time on it as we spend on number, but there are one or two units per grade level devoted to issues in statistics and probability, as well as data activities that are integrated throughout other units. In data, some of the ideas that we're developing include learning what data can tell you about phenomena in your world; learning about representation, and how representation can provide you with different views that help you describe and interpret data; and learning how to show your data and what you've learned from your data to other audiences.

Work in exploratory data analysis is a strong emphasis throughout the curriculum, again, starting in the early grades. As students move up in the elementary grades, we emphasize seeing a data set as a whole that can be described. They're not just individual pieces of data—my piece of data and your piece of data—but the whole set has a shape and a range. There are ways that you can describe it and compare it with other data sets to give you information. As students go through the grades, they learn about some of the statistical measures that are useful in describing data, such as median and range.

Another area of the curriculum that I think is particularly strong is three-dimensional geometry. Our collaboration with Doug Clements and Mike Battista on this 3-D work was very important. The emphasis is on spatial sense and visualization, being able to move back and forth between three-dimensional objects and two-dimensional representations of those objects. Students learn to look at a 2-D representation and build the 3-D object from it, or to take the 3-D model and be able to make a 2-D representation from it.

People sometimes look at our work in 3-D geometry in 1st grade and say, "That's really too hard for 1st graders," but it really isn't. We're starting with kids at a very young age making their own drawings of 3-D models, and beginning to talk about the characteristics of what makes something three-dimensional, and classifying and describing three-dimensional objects. That work gets developed all the way up through grades 4 and 5, where students are making fairly sophisticated drawings and representations and using views, nets, isometric drawings, and other ways of connecting 3-D and 2-D. Three-dimensional models are useful not only in geometry, but also in visualizing structures in number.

The mathematics of change is an important strand in *Investigations*. Students begin to develop ideas about change in informal ways very early. We develop those ideas over time so that students are really beginning to represent and talk about functional relationships by the end of grade 5. They're encountering ideas such as slope—although they may not name it in this way—and what it means in a graph, and how it relates to the situations that it's representing. They're using tables and graphs to

show and describe relationships in, for example, a growing geometric structure. In *Investigations,* the ideas of algebra are being developed across the grades.

Teaching *Investigations*

One way that we want to influence teachers is to help them focus on children's thinking, and on the big mathematical ideas. On the one hand, the teacher's job is to think hard about what the students are doing and saying, and recognizing that children, in fact, have mathematical ideas. On the other hand, the teacher also has a mathematical agenda and mathematical goals in mind for students. As a teacher, you're pulled not only by the students' thinking, but also by your mathematical goals; you're always interconnecting those two things.

Teachers, keeping both of those things in mind, make decisions about which ideas to pursue, which things that students are talking about are important for the whole class to consider, and what questions are important to ask next. "What can I leave until later? What might be unproductive? How can I use wrong answers or faulty reasoning as a way to advance everyone's mathematical understanding?" We hope our materials will help teachers think about these decisions.

Teaching *Investigations* also means working to develop a mathematical community in which students are expected to articulate ideas and question each other's ideas. There is an emphasis on creating a classroom where mathematical argument and justification, or mathematical reasoning, are at the center of daily math work. The establishment of that community, where students can take risks to put their mathematical ideas out for scrutiny, is a really important component of making this curriculum work.

The mathematics background of most teachers has been fairly traditional, and the experiences they usually remember about math are about memorizing rules and procedures. One way we'd like to influence teachers who use *Investigations* is to support them in developing their own conceptions of what mathematics is and what it means to do mathematics. In particular, we want to engage them in thinking about what it means to understand and use the structure of a problem, as opposed to having only a procedural way to solve it—that math is, in fact, about understanding and developing ideas and generalizations about structures and relationships, not just finding answers.

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Supporting the use of *Investigations*

For many teachers, the mathematics content, the pedagogy, and the whole approach to teaching mathematics in *Investigations* is going to be very different than what they've done before. It's not a minor change. Teachers need professional development in a few areas, over several years of teaching, not all at once. Teachers need an overview of the programs' philosophy and approach. They need to understand what the underlying goals and pedagogy are. They need the opportunity to work through many of the unit activities themselves, both to be familiar with the activities and, more importantly, to have discussions about what the mathematics content of these activities is, why kids are doing them, and what ideas they will be pursuing.

Some of the mathematics content is going to be familiar, but taught in a different way. Some of the mathematics content is really new; teachers don't typically have a good background in 3-D geometry, or the mathematics of change, or even work in data beyond a certain point. Learning more mathematics content is a critical

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ongoing piece. That can happen both by working through some of the unit activities as adults, and also through analyzing students' work and talking together about what the mathematics is in that student work.

Another thing that teachers need to work on—again, not all just at the beginning, but in an ongoing way—is studying children's mathematical thinking. Good ways to do that include analyzing student work and using videotaped or written cases. There are a lot of professional development materials coming out now that are very compatible with *Investigations*' approach, that use cases from classrooms to help teachers study children's mathematical thinking.

It's really important for teachers to have professional development experiences with their grade-level colleagues. That seems to be particularly important right at the beginning, in the first two years. Over time, it's just as important for teachers to have cross-grade-level experiences, so that they begin to see the broader sweep of the development of mathematical ideas across the grades. At the beginning, there's a lot to do just in your own grade level: remembering what the next lesson is, getting it set up, and trying to understand and implement this new approach. But over time, you need to know more than just what students are learning in your own grade; you need to have a sense of what happens before they come into your grade, and how the ideas are going to develop as they go on.

Strategies for implementing *Investigations*

We now have quite a bit of experience in working with a range of school districts around implementation of the curriculum, and we've learned a lot from our interactions with those school systems. One thing we know is that gradual implementation is really recommended, by us and by many school systems. And by gradual, that usually means doing several units one year at each grade level, then adding more units the next year, and so forth, so that no teacher teaches the whole thing from start to finish the first year. One caveat: in order to work, gradual implementation has to be clearly defined as part of a long-term plan. It doesn't work to say, "You're going to teach two units this year, and then we're going to see what happens," because then people aren't committed to going forward. People need to understand not only what's going to happen the first year, but what's going to happen over, say, the three years of the implementation; and what kind of support is going to be available, and what they're going to be expected to do each year.

The more we've seen how schools implement this curriculum, and the kind of professional development that's needed to support it, the more it seems that the modular structure of the program is really helpful to teachers and school systems for a very practical reason: it allows school systems to implement the program gradually. In many school systems, where many of the teachers have not taught from a curriculum like this before, one of the things that they really need is to see their students doing exciting and interesting and engaging and intellectually rigorous mathematical work. The modular structure allows them to see their students doing significant work. The length of the modules—four to six weeks—allows students to work in-depth and gives teachers the opportunity to see what it means for students to develop mathematical reasoning, have mathematical ideas, and build mathematical connections. That's very important, and it helps give teachers the motivation and perseverance to go on to implement this program, which is often quite challenging for them.

Leadership

It's critical, of course, that there is a plan for professional development, and that there's a person with responsibility and authority to carry out that plan—someone who has the big view and is on top of what needs to be changed when things aren't working exactly as the system had hoped. Without somebody who is really responsible for the implementation, it's very easy for professional development to end up being haphazard and not very well thought through.

Along with that, there needs to be available leadership and expertise that can provide support for teachers who are starting out doing this. Sometimes that's done by having a group of teachers pilot the materials the year before everybody else is going to start, so they become more familiar with the materials, and have been through it once. Those teachers become leaders, and are able to provide practical support for other teachers, and also have some perspective on what happens once you've gone through the whole unit, or what happens once you've gone through the whole year, which the beginning teachers really can't have. Teachers who are implementing *Investigations* for the first time really need somebody at their school that they know can give them some support. It doesn't even necessarily mean that person leads workshops after school—although it might mean that it sometimes means just the kind of teachers' room discussion that can answer a quick question, or give a teacher some assurance, or provide a practical tip that's really going to be useful.

Parents

It's clear that early attention to parents' education is a critical part of implementing any standards-based curriculum. Parents have a legitimate need to understand what kind of mathematics education their children are getting: what it looks like, what the rationale is for it, and what the goals and results of the mathematics curriculum are. Teachers that we've worked with have found that early, frequent, and consistent attention to including parents in what's going on in mathematics makes a huge difference in the parents' comfort level, and in the understanding that parents have of what's going on in the classroom. That includes everything from phone calls at the beginning of the year, to having parents come in and do mathematics along with their children, to making mathematical work visible to parents in newsletters that are sent home, or in the classroom, or at a math night. Once parents know what's going on, they're able to support what's happening in the math classroom.

At the same time, when teachers are just beginning to teach this program and they haven't had a lot of background themselves, they need help and support in communicating with parents. Until they know the program better and can articulate well what happens for students—until they've been through a year or two—they shouldn't be expected to do all the communication with parents themselves. They need people who do have some of that experience to be organizing and doing some of that communication.

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Technology in *Investigations*

We made a decision as we were developing *Investigations* that we wanted to have significant computer use. We wanted to be able to capitalize on what some computer software can do that you really can't do without it. In particular, we did this in geometry, where dynamic geometry software really allows students to explore relationships that it's very difficult for them to explore with other material. On the other hand, we knew that some schools would not have adequate access to computers, so we didn't want use of the curriculum to be dependent on access to computers.

What we decided to do was develop one unit at each grade level that really integrated the use of computers—in such a way that you really couldn't do the unit well without them. One unit at every grade level uses software we developed that's included with the unit. Those units offer different ways that teachers can actually organize their class to use that software, depending on whether they have a lab situation, or computers in the classroom. But these are not units you could do very easily with limited access, for example, if you only have one computer in the classroom. We knew that some schools and classrooms would not be able to do those units, so we made sure that there's adequate development of geometric ideas in other units through the grades.

Besides those units, there are other uses of software—software that actually comes with the program, or software that is published by other companies—that we recommend as being appropriate for a particular unit. In those cases, the units can be done without the software, but the software very much enhances the experience.

Grouping students

Investigations is intended to be used in a heterogeneous classroom, where the students range from those with significant learning needs to those who have very solid mathematical understanding and need more challenge. The curriculum was tested in classrooms that included that range, and was also tested in special needs classes and classes for advanced students. Our intent was to design problems in which students with different needs, experiences, confidence, and levels of skill could all do significant mathematical work, perhaps in different ways. We don't recommend grouping or tracking students. The evidence from research is that tracking students does not benefit students' learning.

Teachers, of course, have to support the different needs of their students in different ways. Sometimes that means working with a group of students that needs to think about a certain mathematical idea, that needs to work on a certain kind of a problem that's different from another group of students in the classroom. That is the work of the teacher, to figure out the different needs of students, and determine how to meet them. That's different, in our view, from making permanent groups, and labeling kids. Throughout the curriculum, we give examples and recommendations about how to make a game or a problem more challenging, or how to make it more accessible for a student who's having trouble with it.

Impact data on *Investigations*

In general, across the four studies that have been done in *Investigations* classrooms, we see that *Investigations* students do as well as or better than students in classrooms using more traditional approaches. Most of the studies actually focused on number. In particular, the students seem to do very well solving word problems and doing standard straight computation with all four of the operations, and in multiplicative reasoning, in particular. One of the studies also looked at other areas, and in that study, the students in *Investigations* classes also did particularly well on geometry items. A paper summarizing the results of all of these studies is available from our Implementation Center. ■

SANDY JAMES ▸ TEACHER, GRADES 3–4

Implementation

Our major reason for having chosen *Investigations* was that it matched our district curriculum. We wrote our district curriculum based on the national *Standards*, so we were looking for materials that supported the *Standards*. We felt *Investigations* was the best match—not the easiest program to teach, but the best match. We have implemented the full *Investigations* program, although we don't use every one of the units. We choose the units based on the elements of our district curriculum.

Our district has implemented *Investigations* for the past four years. The first year, we used just two units. All the teachers had solid training on those two units. It was expected that we would teach those units and our district assessments were pulled from those two books, so everything was tight. The rest of the year, teachers had the option to teach the other *Investigations* books, but they didn't receive any additional training in those units. Our strength was that we started slowly and had training. The next year, we took a giant leap from two units to eight units per year. It was too large a jump to increase to so many units. We did not give teachers support for such a jump. It was difficult. I think we would do a slower implementation with continued support and training if we had it to do over.

We supplement slightly, but our core curriculum materials are *Investigations*. Fourth-grade *Investigations* doesn't have a specific measurement unit, so we supplement there. In my opinion, the 4th grade also needs more introduction to division. So, where we supplement, it's done to support *Investigations*.

Some of my colleagues wish *Investigations* had more blacklines for homework and more practice sheets. What I have found is that I can quickly generate something on the computer that might be similar to a blackline that meets the needs of my particular group. What I write is what my kids are needing at the time. I find the next year they often don't think the same way and their needs may vary.

One of the big reasons they might not have the same needs is because this program has now been implemented for four years. My kids have had the experience in 1st and 2nd grades of thinking about alternative strategies; what I did with 3rd and 4th graders a couple of years ago would be too easy for these children. In mathematical thinking, they have matured beyond what used to be challenging for 3rd and 4th grades.

Multi-age classroom

I teach multi-age, so I have both 3rd- and 4th-grade levels. The year I piloted *Investigations* was my first year of teaching multi-age. I simply piloted the 3rd-grade level, which is also appropriate for 4th graders. That year was easy. When I began to implement the program fully, I did an integration of the 3rd- and 4th-grade levels units. It was much more difficult than simply teaching one level—it was so challenging because it doubled what I had to know. Rather than knowing eight units, I now had to know 16 units.

We've written guidelines on how to integrate for multi-age. The different math strands of the curriculum are covered in units for both grade levels. We do an integration of 3rd-grade and 4th-grade books that are on the same subject matter. We take the 3rd-grade *Landmarks in the Hundreds* book and integrate it with the 4th

Sandy James teaches at Madison Park Elementary School in Phoenix, Arizona. She began her career as a K–8 music teacher, taught 3rd grade for six years, and for the past five years has been teaching multi-age grades 3–4 classes. She teaches in the classroom half-time. The other half of her time, she serves as a district math teacher leader, supporting teachers in the classroom and facilitating grade-level and district-level planning meetings.

Madison Park is one of seven schools in a small urban school district in Phoenix. The district serves approximately 5000 students in grades pre-K through 8. The student population is diverse, including 13% of students who require ESL support, 41% of students who receive free and reduced lunch support, and many students from high socioeconomic and professionally-employed parents. The district has been using *Investigations* in grades K–5 and the *Connected Mathematics Program* in grades 6–8 for four years.

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grade *Landmarks in the Thousands* book. That's pretty easy to do. We have a multiplication and division unit in the 3rd grade, *Things That Come in Groups*, that is integrated with the 4th grade multiplication and division unit, *Arrays and Shares*. The 3rd graders are doing some 4th grade activities, but they're doing them superficially. When they revisit them as 4th graders, they say, "Oh, that's what you were talking about last year." I watch them be confused about some of the stuff in 3rd grade, because it's pretty big stuff. Then in 4th grade, it's a cinch for them—so then they go deeper into the understanding of the mathematics.

I see 4th grade as the standard for 3rd/4th multi-age and the kids have two years to accomplish that. Some kids accomplish it quickly, and some accomplish it more slowly. But, by the end of 4th grade, they have risen to the standard and many have gone beyond.

We have had more multi-age classrooms in our district, but because of the challenge of integrating two levels of *Investigations*, we have lost some. Teachers were overwhelmed, so they pulled out and went to single-grade. Now that they're more experienced with the program, some want to go back because they see the strength of a multi-age setting in doing this kind of teaching.

Meeting needs of all learners

One of the equity issues we're concerned about at my school is learners of English as a second language. In *Investigations*, we talk so much about what we're doing, and sometimes that's very difficult for our ESL students. We do expect our students to communicate a lot, sometimes verbally and sometimes in written form, and that's been a concern for us with our ESL students. Often, we have them working with other students who are fluent in both English and their native language. Before *Investigations*, I found my kids, no matter what language they spoke, could manipulate the plain old numbers and they could borrow and carry. This program goes far beyond that, so sometimes some feel unsure and get scared.

On the other hand, the many hands-on activities and visual support are good for our ESL students. It's also helpful that the kids are given the options to explain their thinking in illustrations, diagrams, or charts.

Mathematical goals for kids

I want kids to be able to reason mathematically, to use logical thinking whenever they're approaching mathematical situations. I also want them to be able to communicate what they're doing, so they can problem-solve with each other. We work a lot on that, so they're not just isolated workers, but are able to communicate what they're doing to their partner, to a small group, or to the whole class.

I've learned in the last few years that kids have a lot of mathematical understanding that they've been acquiring since they were small children. They come to school in kindergarten with a basic understanding of mathematics in the world around them. We don't teach our students everything they learn. They assimilate all they've experienced into their own unique system of understanding—that's how they learn. I used to think I had to teach my kids every little thing they'd ever need to know about mathematics. Now I realize I'm just a little cog in this big wheel of their understanding of the world.

The way I used to teach, using my adult thinking to tell students about math, was often confusing and didn't make sense to them. Students often wouldn't trust their

own understandings anymore. I try not to do that now. I try to let their way of looking at the world mathematically be the thing they trust. My job is to build on the understandings they have in place.

Mathematics

Investigations covers number sense in such depth. The program emphasizes relationships in number, like skip counting, and uses visual supports, like array cards, to aid understanding. It's not just numbers out there hanging as abstract things; the kids have a visual understanding of what numbers represent. Children learn how to manipulate numbers, how to take numbers apart and put them back together. We don't do traditional computation. Some kids have been taught it by their parents so it comes up as kids are sharing strategies. But most kids have developed alternative ways of computing that are just as efficient and make more sense to them. We have samples throughout the room of different ways that kids do calculations. Sometimes we name a strategy after students so when another kid understands how to do "Nicole's" way, they connect, "Oh, I remember how Nicole did that and I can do that with this problem." Kids model for each other all the time.

Geometry in *Investigations* is so broad-based. There are many hands-on activities and visual supports. There are activities using manipulatives such as geoboards and snap cubes that are used for geometry as well as support for number concepts. When students use fraction pieces they've made themselves, they see the relationships and equivalencies. When they start with a whole and make eight pieces out of it, they really know why it is one-eighth.

Probability and statistics are pervasive throughout the books. We look at what's typical, what the range is, where there are clusters of data, where there are holes without data. We graph data about students in our class. We ask many questions to stimulate the kids' thinking about the data we've gathered and what it really means. It's not just a bar graph of things that happened to somebody in some other part of the world. The data is real and relevant.

In the 4th grade, measurement is weak, but in 3rd and 5th grades, it's strong. When you look at the whole program, measurement is not a weakness. The measurement in the program gets at the reasons for the conventions of measurement. The kids work with both standard and metric. They relate units of measure to themselves at a personal level: "What is an inch? What do I have in my body or in the classroom that's an inch? A centimeter? A foot?"

Challenges for teachers

I would say one of the hardest things for most teachers with this curriculum is letting the students develop their own strategies. I've had teachers see charts of my kids' strategies for computation and try to teach those strategies to their class, rather than letting them come from their own kids. It's hard to trust students' mathematical thinking. We're used to having to pound things in rather than pull things out. That's a real challenge for teachers, to step back and wait for kids to develop their own understandings. But, the intent of the program, I think, is to pull the mathematics from the kids, not to give it to them, not to give them an adult understanding of mathematical thinking, but to let their own thinking develop. Two of our books are *Mathematical Thinking at Grade 3* and *Mathematical Thinking at Grade 4*. That means the focus is on kids' thinking in that grade—not ours.

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I find it helps to ask kids intriguing questions—to stimulate their thinking with questions like, "Can you prove that to me?" or, "What were you thinking when you went in that direction?" As teachers, we need to ask, to listen, and then we're ready to support learning.

When teachers step back and let it happen, it's very powerful. When we let kids do it, all of their reasoning, thinking, and logic come into play. We let them talk to each other about how they're solving a problem, or we let them argue about what's the most efficient way to do it. When somebody has a strategy that works for them, they want to share it with the whole class; they're eager to get up to the overhead and try to explain it to the rest of the kids.

But stepping back is the very thing that is the most challenging for teachers to do. We've been information-givers and answer-givers for so long, that when we're expected to let children figure things out on their own, it's tough to do. When we don't look at the big picture of how kids think mathematically, we get scared and think, "They're not going to learn this unless I start telling them how to do it."

Time

This program takes an incredible amount of time for the teacher. The teacher has to read the lesson, read it over again, and then often talk with somebody and ask, "What do you think this means?" I also take monitoring notes on students so I see where the kids are and where they need to go next. It takes time for reflection to sort out all of this information. It's certainly nothing like turning to the next page, telling the students to do the work, and then correcting it together. As a teacher, preparation time is greatly increased.

Another challenge for me is to have enough time to include everything. I would like to have two hours a day doing math—then I would feel like we've maybe had enough time.

In the first year of piloting and the first year of implementation, I did almost exactly what the books said to do. Now after teaching the program for a few years, I'm much freer with my timing, my pacing, and what I do first. I have the big picture of what's in the unit and what's in the whole curriculum. But at first, because the program has everything so detailed, I did exactly what was written.

Pacing

One of the challenges of the program is that it's so comprehensive we can't get through everything. But, on the other hand, if they didn't include so much, it would probably have too many holes. I don't mind that the program has books I don't get to now, because I'll probably need and want them at some point. My first year, I got through eight books because I thought I had to for piloting purposes. Did my kids understand all of what we went through? No. Was I basing my teaching on what they understood? No. But I got through the books. Now my pacing has to do with assessing where the kids are. By assessing their understanding I know what the next learning steps need to be for my group as a whole, and for my students as individuals.

At times, I can skip right over stuff we had to work really hard on a couple of years ago. But then I get to something else and I notice we need to do more than is provided in the curriculum. So I plan around where they are and what they need at that point. I can't say that I get through as many units as I did that first year—but I do know we get to the math in more depth now than we used to.

Teacher support

The first year of implementation, all district in-services were on math. Teachers had a great deal of support. There was a summer workshop, and study groups were set up during the school year. Our second year of implementation, we took a giant leap to doing all the units, and we didn't have the in-service available to us. The leap was too big. There's an ongoing need for in-service because this is a different way to teach math.

We recently received a National Science Foundation grant to support teachers in implementation. The grant provides funds for grade-level planning sessions, classroom observation and dialogues, summer workshops, and university coursework for all teachers of mathematics.

Assessment

Investigations has a combination of teacher checkpoints and embedded assessments that definitely work well. I get a great picture of the strengths and weaknesses of my kids and what continues to be challenging for them.

The program doesn't include structured preparation for standardized tests. We spend a lot of time encouraging alternative strategies, and these take time to develop. *Investigations* definitely stresses that kids should understand and be fluent with number facts and computation, but they don't present a format to assess fluency and efficiency. So, the one kind of assessment I do that's not a part of *Investigations* is a page of problems in a similar format to standardized tests, having kids focus on doing the computation in an efficient, timely manner. I feel my students need this format and practice to prepare for testing.

Parents

Some parents have felt threatened by the change in our mathematics teaching practices. They were so worried that their children weren't going to know their basic facts, that they wouldn't be able to do traditional mathematics. I think they're finding their children can, in fact, do those things, plus more. I think we're getting more support from our parents as time goes on.

We're not seeing a problem with our standardized tests. In fact, we're seeing some improvement in the scores. What the standardized test measures isn't all the students are learning anyway, but it does speak in numbers and percentages that people can understand.

Administrators

Administrators haven't had as much training as they need. Most of our administrators are language arts people and they don't have an in-depth understanding of reformed math. They trust the math experts and teacher leaders, believing this is a good program. They are experienced in a reformed literacy program, so when they observe math classes they're seeing a similar philosophy, and that's helpful.

This past year we have been involved in a research project involving administrators and teacher leaders. We have worked together to deepen understanding of reformed math. Our district administration is very supportive and has high expectations of program implementation. ■

SANDRA ELDRIDGE ▸ TEACHER GRADE 2

Sandra Eldridge has been teaching *Investigations* for two years, as a 2nd-grade teacher. Sandra taught kindergarten for 16 years, and 2nd grade for five. Her school, W.J. Clinton Elementary School, is a speech and technology magnet school in the Pulaski County Special School District, in Sherwood, Arkansas. As a county school district, Pulaski serves both urban and suburban populations, and has students from a mixture of socioeconomic backgrounds.

Goals for children

I want children to have a better understanding of math. A lot of children hate math because they don't understand it. The way we used to teach math, there were just one or two ways to solve a problem, and if you couldn't solve it that way, you were usually unsuccessful in mathematics.

Well, one thing I've come to realize is that when we teach reading, we teach children many strategies to learn how to read. The thing I like most about *Investigations* is that we're teaching them more than one way to solve a problem. They explore and figure out what's best for them. It's not just a teacher dictating to them that this is the best way to do it. In experimenting with the math problems, working in small groups, and sharing strategies, children soon find out a way that's best for them. I see children having a better understanding of number sense because of their shared ideas. I've learned, too, that sometimes the best teachers are the children themselves. I've found myself teaching a lesson and thinking that they still don't understand. But if I get a child to explain their thinking, it's like, "Oh, okay. I know what you mean."

I try to meet the needs of all children. So, I give them a visual, hands-on approach in all subject areas. I feel that if children are more involved with the learning process, they probably will retain more. I know I retain more from actually doing the activity myself.

Why *Investigations*?

We are fortunate to have a principal and math specialist at our school who saw a need for us to use different approaches to teaching math. We also saw that children weren't scoring where we thought they should be on national standardized tests. We wanted to find another way that we could teach to bring up children's test scores. We felt that if children were more involved in the learning process, then that might help. *Investigations* believes in student involvement.

Two years ago, our math specialist asked me to attend TERC training. We attended the training and were just sold on it. So we came back and implemented the program the first year, along with another 3rd-grade teacher who attended the training with us. I began sharing ideas with other second grade teachers, and it turned out that they wanted to use those ideas in their classrooms. So, the 2nd-grade teachers wanted to implement the program. It grew through word of mouth. More teachers decided that they wanted to try the program, so we had workshops where they could get a feel for what the program was about. Most of the teachers thought that this was a program that they wanted to implement. Eventually, we voted on the program, and W.J. Clinton decided to use *Investigations*. Everyone in the school would teach the curriculum in grades K–6. We had training last summer for a week, all-day training for a whole week. Our math coordinator for the school district and our Title I math teacher here at Clinton Elementary did the training.

In our district, schools make the decision about which mathematics program to use. Since this is a new program, when it was time to do textbook adoption, we had the option of using this program or adopting math books. There is one other elementary school in the district also using *Investigations*. And this week, we have another school coming to visit and ask questions about the program because they're considering implementing *Investigations*.

Strengths of *Investigations*

One of the strengths I see in the program is that it allows the teacher to be more of a facilitator. You try to ask the right questions to get the children to think more about how the problem should be solved. You also use small group discussions that allow children to interact and share ideas.

Another big strength is writing about math. It's so hard for children to put down their thoughts concerning any subject area, but especially math. But now that this is something they do on a daily basis, they're getting used to it. It's becoming very easy for them. You know, if you don't need to do something, then you don't have a reason to grow in that area. Never before, in any math program that I can remember, did children have to write down their thoughts as much as they do in *Investigations*. You can see your mistakes, too, when you write out your steps.

Another plus for me as a teacher is the timeline that is set up to keep teachers at a good pace in the program. It helps you, as a team, to kind of stay together. At the beginning of the school year, we had what we call a "follow-up," a meeting with the schools in Pulaski County and surrounding areas that are using *Investigations*. It was amazing how many teachers were on the same lesson.

Basic approach of the curriculum

Investigations helps children to develop an understanding of math and number sense. Another key to the approach, I think, is that children are learning different strategies and ways of solving math problems. It's causing children to be creative thinkers, better at solving problems.

Here's one lesson that we did. At the beginning of the year, we were doing the "magic pot"—where every time the lady put something in the pot and pulled it out, it would double in number.[1] Later on in the curriculum, we had a lesson with strings of numbers—they had to add strings of numbers and some of them dealt with double facts. It was like 8 + 8 + 6 + 6. Well, they already knew their doubles, so they did 8 + 8 is 16 and then 6 + 6 is 12. They used that approach, based on the knowledge and concepts that they had already learned to solve strings of numbers.

"Turn Over Ten" is an activity that helps children learn sums of 10.[2] They look for sums of 10 in combination when they're adding strings of numbers. When they have to add a string of numbers, they find double facts, sums of 10, add those up together, and solve the problem using those strategies.

Using mathematical tools

Investigations helps children learn to use different mathematical tools in solving problems and when is best to use them. It allows children to find out, for example, when the calculator is faster than using mental math. There's a particular investigation that proves that people using their heads would solve the problem a lot faster than the ones who were using calculators. The children were just devastated. We talked about the fact that, from this lesson, you need to learn that sometimes it's faster to use mental math. The curriculum will teach children not only how to solve a mathematical problem, but also which strategy works best, which doesn't, and that we should try using different strategies.

Investigations helps children learn to use different mathematical tools in solving problems and when is best to use them.

[1] This activity is called "Two of Everything" and is in *Coins, Coupons, and Combinations* (grade 2).

[2] In *Coins, Coupons, and Combinations*.

The instructions for the teacher are very clear. With the teacher's guide, there are a lot of teacher notes that are scattered throughout the sessions.

Our class is now at the point where we're finding out that sometimes it's a lot faster not to use cubes, but to use numbers instead. They're just learning what tools are necessary to solve problems, and when to put those things away. I think that's a life skill.

Mathematics

Investigations does a good job with problem-solving, computation, combinations, classification, and data. It does an excellent job with geometry. As a teacher, that was my weakness. Each time I go to a training session for *Investigations*, I have a better understanding of geometry. In the math books that we used in the past, there were just a few pages of geometry, and you didn't get into understanding it. *Investigations* covers much more than most math books. Children are working with geoblocks, and filling in spaces and shapes, and understanding, for example, a trapezoid, a hexagon. Parents are really amazed with the geometry activities we do, and sometimes they have questions on how to solve the problems. They can't believe that their children understood what shapes it took to make another shape and what shape it took to fill in a space.

Setup of materials

The books are set up so that you know beforehand exactly what you need to do to present the lesson, and then the lesson itself is set up so that it ties into the next concept of that unit. We were just talking today in our 2nd grade team meeting about how we know which student worksheet to use, and what materials we need to have for a particular lesson. We know when we teach the lesson whether we need to have small groups or start off in the large group. When literature needs to be ready to use for a particular lesson, we know what books we need to read. All of that is outlined at the beginning of a Session.[3] You don't have to decide what materials to use; it's right there. It cuts planning time—you plan and gather your materials probably in half the time that you did before.

The instructions for the teacher are very clear. With the teacher's guide, there are a lot of teacher notes that are scattered throughout the sessions. I think they were put in there to give you some idea of how children respond to the lesson and what questions you can ask, which is good. It seems like a lot to me now, but for beginning teachers in the program, I think they would like that. I think it would help to pull the important facts about a particular lesson and reduce it. I have told teachers, "It's like your Bible until you get used to the format. You find yourself carrying it everywhere you go."

Support

Our math coordinator wrote a grant so that we would have the funds necessary to implement the program. We realized that we had to make a lot of materials, and we wanted to use those materials for a number of years. We needed laminating paper for manipulatives and literature. The grant also pays for the workshops that they've established for follow-up, where we can share with other teachers, and the workshop that we had prior to implementing the program, for all teachers who were going to teach the program this year.

I don't suggest that any school district should begin the program without some form of workshops. You need at least a week to two weeks of intensive workshops

[3] Within an investigation, the activities are organized by class "session." A Session is designed for a math class of at least one hour.

on the curriculum, so that teachers will have an understanding of how to implement. Teachers need to actually do the hands-on work to experience what children will be experiencing, and see how children are thinking as they work through the process. If a school district doesn't have staff development, they're setting themselves up to fail.

Parents

Some parents say their children are enjoying math more. I've heard parents say that when they've planned doctor's appointments, their children don't want to miss math class. Even though parents don't understand it, they're hearing that their children feel good about it. I think that is probably one of the biggest pluses for any program—for a child to say that they want to be part of it, that they don't want to miss it.

Parents are saying that they're seeing their children working math problems at home. There are activities, homework activities that you send home and they have to work with and play games. At first, parents didn't understand the importance of those games. But now, as they work with their children, they can understand.

Assessment

Assessment is something that we're discussing as a school. As a classroom teacher, you're always looking for that ready-made assessment. We feel pressure to have some form of assessment where you give a grade. A lot of parents also want that piece of paper that says this is a test and this is what your child scored. They want to know how their children compare to other children.

In some other programs, there's a book with assessments. When you get into a new program and it's not there, you miss it. I think that's a weakness of this program—there aren't ready-made assessments.[4] We've worked together as a team and have come up with assessments based on student work pages that are in the program.

One thing I'm doing this year is timed tests to see if children are learning basic addition facts. A lot of the parents were concerned about whether children are learning their facts. I've pulled tests from the ends of chapters in our previous math book to see if the children I'm working with were scoring as well as other children. I've been very pleased with the results.

All learners

Investigations meets the needs of all children, because it works on the child's own approach to math. The child's giving me, the teacher, what he or she knows. My role is to ask the right kind of questions to get him or her thinking about how to solve the problem. It's not my way—the teacher's way—of doing the problem, it's the student's way.

Everybody is learning based on what they're bringing into the program. As the teacher, you facilitate and ask the right questions to help students to get a better understanding and give you some feedback. I think it works for all learners, for all ethnic backgrounds, both genders. For some students, we're still at the point where we may use more manipulatives than with other students. But they're going to eventually move from manipulatives and be able to solve problems by thinking about how the problem should be solved. They will have that number sense built in. ■

Teachers need to actually do the hands-on work to experience what children will be experiencing, and see how children are thinking as they work through the process.

[4] The authors of *Investigations* are currently developing a book of end-of-unit assessments, which will be published in 2001.

DEBBIE SHEIN-GERSON ▸ ELEMENTARY MATHEMATICS COORDINATOR

Debbie Shein-Gerson has been the elementary mathematics coordinator in the Brookline, Massachusetts schools for 15 years. Before that, she taught for 11 years, in all grades K–5 and as a Chapter 1 teacher.

Brookline has been using *Investigations*, *MathScape*, and IMP. The district took part in the field testing and development of *Investigations*. Elementary schools in Brookline have students K–8. The community is considered urban/suburban, with parts that are very urban and parts that are quite suburban. Over 60 languages are spoken by students; there are bilingual programs in Spanish, Russian, Chinese, and Japanese.

Background

Before we implemented *Investigations*, Brookline used a curriculum called DMP—*Developing Mathematical Processes*. The first purchased curriculum of any of the subject areas was math, and it was DMP, which was quite ahead of its time, in that it wasn't a traditional textbook. It was a series of units—even more units per grade level, I think, than are in *Investigations*. For those days—the early 1970s—it had quite a strong strand of geometry. DMP was actually developed by Mary Lindquist and Tom Romberg while they were at the University of Wisconsin in the late "60s. We used DMP through the life of the program, until it was no longer available for purchase; then we xeroxed units ourselves, because there wasn't another mathematics program we were willing to use.

Brookline has had a strong history of professional development around mathematics. We've had the money over the years, prior to the selection of *Investigations*, so that teachers could have quality, state-of-the-art mathematics training. We are fortunate to have teachers who are quite talented. The professional development, and the quality of our teachers are two reasons we have been successful implementing the *Investigations* mathematics program.

Why *Investigations*?

Our primary goal is that all children will be successful in mathematics. We are committed to educating all children. Wherever students are when they come into the system, it's our job and responsibility to take them to where they need to go. We wrote learning expectations for all the curriculum areas, in all the grade levels, before it was fashionable. The math learning expectations are based on the NCTM *Standards* that were written in 1989. I took those *Standards* and basically rewrote what they said, and then, for each grade level, I specified what should be taught. The Brookline Public Schools has learning expectations for grades K to 8 in mathematics. We used these learning expectations to guide our selection of a mathematics program. We had many criteria, but the first one was what program would best match our learning expectations.

We also had other criteria: "How will a mathematics program meet the needs of children who are not grouped and tracked?" We cared about the mathematical content, but we also cared about the instruction. We had been using a curriculum for years in which children worked in pairs, in groups, and at stations. So we asked, "How would a new program promote the instruction that we believe should be happening for children?"

You can tell that *Investigations* was written by the same group of people from start to finish. It's like this beautiful puzzle that's been put together. There's an extraordinary spiral in the program with the big ideas around geometry and number and data.

I think that the teacher's guides are fairly easy to read. The bonus is that they've been written to also provide professional development. Teachers are learning as they're teaching. Teachers aren't being talked down to—it's written for the level that my teachers can use and then deliver the instruction. That's significant. A lot of my teachers say to me, "I used to solve the problem this way, but now that I'm

teaching *Landmarks*, I do it this way."[1] They've learned more mathematics while using the *Investigations* math program.

One strength is that, at all grade levels, there are consumable booklets that students can write in. Some people may see that as a weakness; I think that's a strength. The format and the layout were similar to DMP and accessible for students. It's child-friendly.

Invented algorithms

In my district, having children use invented algorithms for computing is something we are still thinking about. We're trying to deal with this issue in a way that is philosophically aligned with *Investigations*. We are grappling with what to do with students who aren't inventing an efficient algorithm. In a traditional program, teachers would just teach the traditional algorithm. Students might learn "how to do it" but not have any deep understanding of the process. Another question is, how do children develop fluency with math facts, and fluency with algorithms? The authors of *Investigations* know that these are issues educators are struggling with. I don't think it's necessarily that you have to teach the traditional algorithm. What we do in Brookline, because we have such a diverse population, is to teach the different algorithms that the students in the class bring from their home countries. We say, "There are all these different algorithms." We're asking students to have more than one algorithm, more than one way to solve the problem.

The program teaches number in a way that makes sense and is useful and practical in the real world. To me, its strength is that it gets at understanding, as opposed to memorization.

Reaching a spectrum of needs

Investigations supports more students, and more students are getting a greater range of mathematics than ever before. One of the program's greatest strengths is that all students are learning more geometry and statistics. Most students can participate in most of the activities, because the activities are open-ended enough to allow students to be part of a working group. The layout of the student sheets is very clear; they are not filled with a lot of directions or pictures. If a student is not reading on grade level, they can use these materials, which is a tremendous strength, and having a below-grade-level reading level doesn't interfere with learning the mathematics. *Investigations* taps into all the different learning styles. We've seen students' strategies for solving problems vary with students' abilities.

As with any other program, for the extreme ends of the spectrum, teachers need to make adjustments. For the students who clearly are special ed, we need to make further modifications. For the brightest students, the arithmetic strand is not challenging enough. But I don't know of a mathematics program that has been written that meets all the needs of all students. With *Investigations*, the range of children who can be successful as learners of mathematics is broadened.

Implementation and support

There are two ways to implement this program. One is, you just do it. You just say you're going to do it. The reason you might do this is that it could take you forever—you couldn't possibly implement just one book a year.

We had many criteria, but the first one was what program would best match our learning expectations.

[1] Refers to units named *Landmarks in the Hundreds* or *Landmarks in the Thousands*.

The key to implementation is to train teachers around the mathematical content before or during the teaching of these units…

The other way is to implement it partially. If you're not going to do the whole program all at once in your implementation year, then start with the geometry and data strands first. I wouldn't do the arithmetic strand first. I feel very strongly about this. That's the strand that needs the most continuity, and that's the one that needs the most professional development around invented algorithms. Now, the other strands are also hard for a lot of teachers, because teachers have never been taught some of the mathematical content. The key to implementation is to train teachers around the mathematical content before or during the teaching of these units, because in order for them to feel comfortable, they have to have some basic mathematics knowledge. Unfortunately, in geometry and data, too many teachers don't have the background to teach these topics until they have had professional development targeted to these content areas.

Whatever way you decide to implement the program, you need to have a plan that's clearly stated. Then the teachers know what to expect. You need to provide in-service before, during, and after. That takes a lot of time. You can't do this if you don't have professional development. I knew it was going to take a lot of time, money, and training. If you don't have the professional development, I don't think the program can succeed—except for the best and brightest teachers, who would make it work.

Challenges for teachers

In the beginning of our first year, we asked the 3rd graders to write about their thinking. It was difficult for both students and teachers. Students had never been asked to do that before. Teachers weren't sure students could do it. So what we actually did was, I went into classrooms and I just did a whole lesson, the same lesson in different classes, and then collected all the student work. Then I had a staff development meeting, and I made all the student work into overheads. The teachers had copies of the same. I said to them, "Okay. Does this look like your class?" They said, "Yes." And then we talked about how you can get students to write more. I worked with the language arts coordinator, who validated that, one, it was important for students to write in mathematics class, and two, students at this age could do it. I collected my own data and said, "Okay, it is a problem. But I'm not saying it's too hard, that we're not going to do it." We modeled what we wanted students to have in their writing, and subsequently, students were able to write about their thinking.

After that first year, when students were told right from the get-go that they had to explain their thinking, it became less of an issue. The teachers would say, "It's so much easier this year, because we're not fighting with the students about whether they have to write about their thinking." There are two big ideas in this program—one, having class discussions, and two, students writing about their thinking—that need to become part of the classroom culture. Eventually they do become part of the culture, but it takes time.

Teachers need a lot of support, especially in their first year of teaching *Investigations*. We talked a lot about what it feels like to teach when you have to hold the teacher's manual in your hand. As a teacher, you've got to make a decision the first year that that's what you're going to spend a lot of time preparing for, because you have to read the teacher's manual carefully. You've got to read those Teacher Notes, the Dialogue Boxes.[2] Teachers have to make a commitment to do that.

[2] Teacher Notes are boxed-in sections of text in the teacher materials that provide teachers with information about mathematics content and about students' development of mathematical thinking. Dialogue Boxes are samples of what a classroom discussion around a problem might look like.

In the beginning, there was nothing that looked familiar. At one point, I asked teachers, "What was your most successful lesson in the unit?" What turned out to be their most successful were the kinds of lessons they might have already done before using *Investigations*. It was usually something like reading a literature book, and doing a mathematical activity. I realized it was the familiarity of doing those lessons that made it successful. All that isn't an issue after the first year, because teachers have taught the lessons at least once, so they know what to do and what to expect. The real benefit of this, of course, is that it's a real growing experience for all teachers. And it builds community and collegiality, because you all have come through this process together. So it's a very powerful experience to have implemented the program together.

Pacing

Initially, there was a coverage and pacing issue. Teachers couldn't get through the whole program—there are just too many units at certain grade levels. In the first year, in kindergarten or the 1st grade, teachers could teach all but one unit. In 2nd grade, teachers could probably teach six units, and in 3rd grade, probably six units, also. Fourth grade, there are way too many units. Because our students have been in the program for so long now, what I'm doing is looking to collapse some of the units together. Fifth-grade teachers are saying, "All students are coming in now with a stronger sense of number." It's exciting and rewarding to hear this. So we are collapsing some units together, because at the point we are now, we think we can teach concepts in less time because the children have a more solid foundation.

Assessment

The first year using the program, the teachers didn't realize the assessments were in the program. It was hard enough just to do the activities. We've used the book *Beyond Arithmetic*.[3] I have a joke now: we're beyond activities. At first, teachers could just do the activities, and they were so exhausted from the whole experience of getting it done and getting students to do the lesson that they couldn't see the assessment component of the program. It was only after a year or two that teachers began to say—because they felt more comfortable—that when students were actually working in groups, they could walk around and watch students at work. It was only then that teachers were really paying attention to the assessments, and really reading what the program says you should be looking for, and seeing where and how their classes matched up.

Now, what teachers find very scary is that, before, they didn't know what they now know from the assessments. Some of my best teachers say, "That's really scary. So what was I doing before? And now that I know, I'm trying to figure out what to do with that information." We're at the point where the teachers will say, "Okay, this student could do this. Another student understands that. What do I do next? What do I do tomorrow?" No program can tell you that. But this puts in place that first big step, which is getting teachers to look at students' thinking, and that's the first step in using assessment to inform instruction.

There are two big ideas in this program—one, having class discussions, and two, students writing about their thinking—that need to become part of the classroom culture.

[3] This book, by several authors of the *Investigations* curriculum, focuses on how and why mathematics teaching and learning are changing. It is available separately from Dale Seymour Publications. (1995; Order #DS21259; ISBN 0-86651-846-0).

What we have learned is that the visual models of some of the number ideas are very powerful for all children.

What's working well and what's challenging for students

During the first year only, those students who saw themselves as strong math students felt a little uncomfortable, because for the first time, they were being exposed to topics that they had no familiarity with, particularly around geometry and data. It was wonderful, because it meant that we were challenging them now. My own son, who was gifted in math, came home one day and said, "Mom, do you know that square numbers are square?" And I said, "Of course they're square. What do you think they are?" He said, "No, they're really square." In class, he had to build all the square numbers. And he didn't realize until then that they formed a square. There is an activity where students cut out arrays for multiplication, which some teachers think is incredibly tedious. What we have learned is that the visual models of some of the number ideas are very powerful for all children.

The way manipulatives are used to construct and figure things out meets a lot of students' needs. The hands-on experiences—for example, the activities with geoboards for fractions—are very powerful in helping students to really understand the area model of fractions. For some students, using manipulatives is the best way for them to learn a concept. And for others, like my son, it uncovers a concept that isn't necessarily apparent.

Classroom discourse

One thing that is still challenging for teachers is conducting the classroom discussions. The Dialogue Boxes in the program help, by giving examples of actual classroom dialogues, but teachers really have to know the mathematics content to be able to respond to some of the students' strategies and ideas.

Having classroom discussions was different for students as well as teachers. We had to build up students' endurance and listening skills. I spent a lot of time in classrooms having discussions with students, and then I gave teachers some strategies to use in their classrooms. There are things you can do when students come together as a group to share: students can practice before they present to the group; they can have visuals to enhance their presentation. We learned a lot of the mechanics of that, and some of the mechanics of how to run a classroom discussion. We're still grappling with both. I find people still have difficulty with the discussion component of the program. But students get better at that over time, and so do teachers.

Parents

My advice is that you have to do a lot of communicating and working with parents, teachers, and students, telling them that this is different, and why it's different, and the rationale for why the school system is implementing a new mathematics program. The reason is to promote mathematical literacy and make mathematics accessible to all students. I think you have to communicate to parents that mathematics needs to be different from what they learned in school, because we are preparing students for a different workplace. We had many information meetings for parents, family math nights, and PTO breakfasts, to keep parents informed. We taught the games in the program to the parents. There's a tremendous PR piece that is necessary to ensure successful implementation and acceptance by parents.

I've written two letters to parents about the curriculum. One explained our selection and implementation process. As a parent—I am a parent, too—I want to know that there is a well thought out implementation plan. I thought about what I would

want to know, and I wrote this letter. Then I gave it to several parents to read before I sent it out to all parents in the community, and asked, "Does this answer all your questions?" So it's a very informative letter.

I also wrote a letter to parents about our philosophy of computation and learning basic math facts. The letter said that automaticity is important—children need to know math facts—and that whatever program a school system uses, there is never going to be the right amount of practice for all children. In *Investigations*, students build mathematical understanding before they memorize math facts, as it should be. What I ask parents, and obviously, teachers, to help with is the practice. It's like learning an instrument: you have to practice to get proficient. So the letter gives parents ideas on how to help their children at home, and lets parents know, for example, that there's more than one way to solve a problem.

Principals

Often, principals do not have a strong background in mathematics. When a school system adopts a new mathematics program, there needs to be some professional development specifically targeted for administrators. Administrators need enough information to be able to communicate to parents and assist teachers through the implementation process.

Principals need to articulate what level of knowledge they need, in order for them to do a good job supervising and evaluating teachers. They also need a familiarity with what are the big ideas that students need to know and be able to express. I've been working with a first-year principal in Brookline, and she's been visiting classrooms. I've told her, "What you need to do is ask the teacher what lesson he or she is going to teach. You need to take the teacher's manual, and you need to look at the lesson. Only then can you evaluate how well the teacher is teaching the lesson." Often, I go with principals to observe teachers teaching *Investigations* units, pointing out specifics that they should look for. They should pay attention to the questions teachers ask, engagement of students, teachers' mathematical knowledge, and comfort level with the math content.

For the most part, my administrators are very supportive. Together with teachers, administrators have learned more mathematics. They have experienced the implementation of a standards-based mathematics program. I think they would all agree that this is a better way for students to learn mathematics. ■

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